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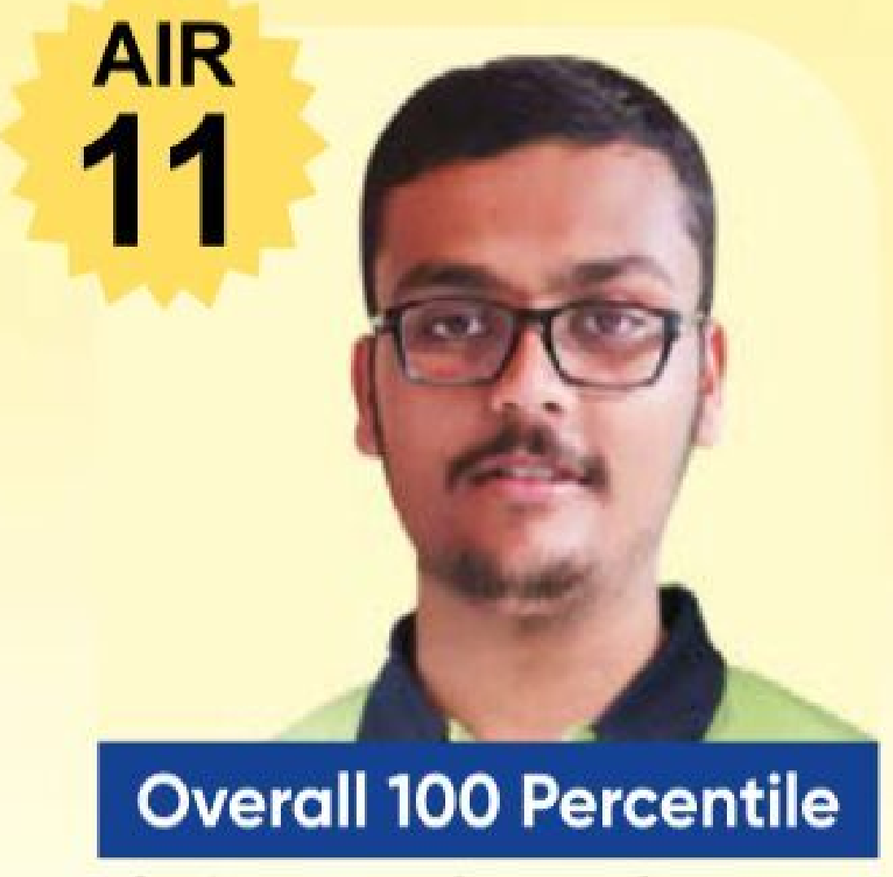

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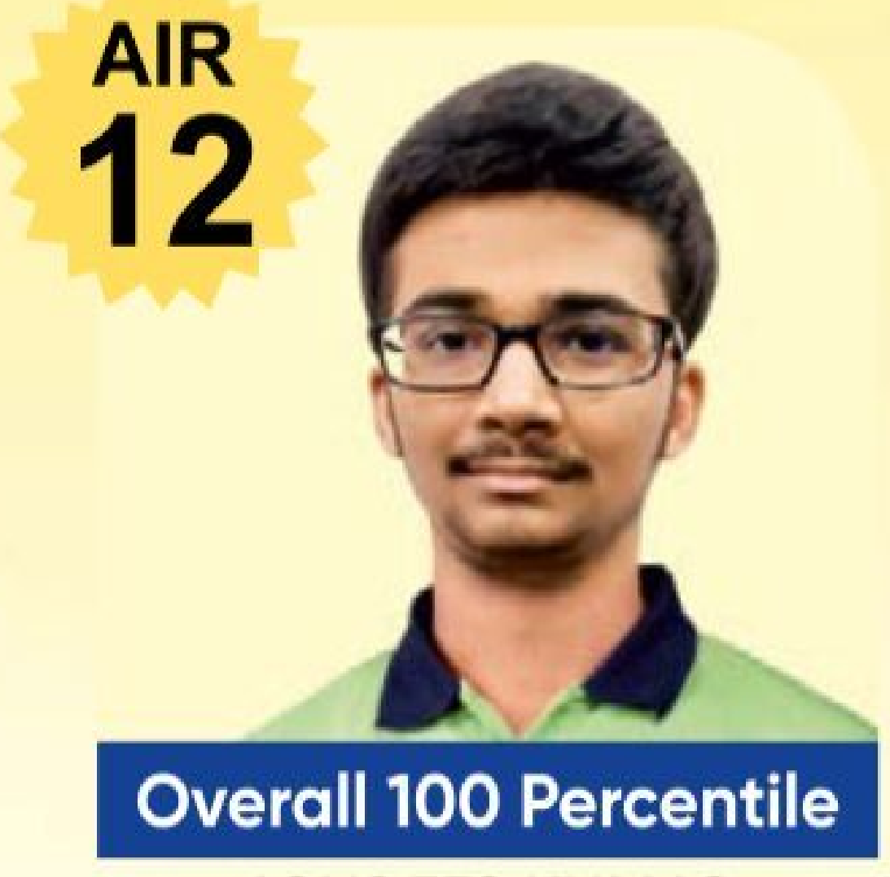

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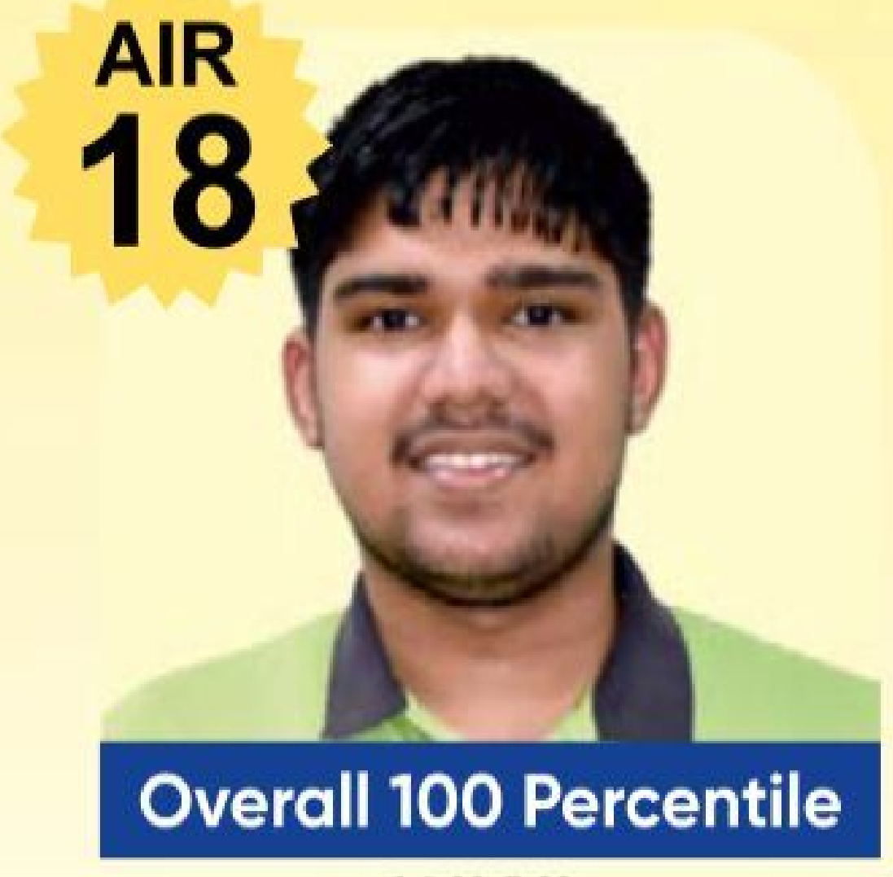

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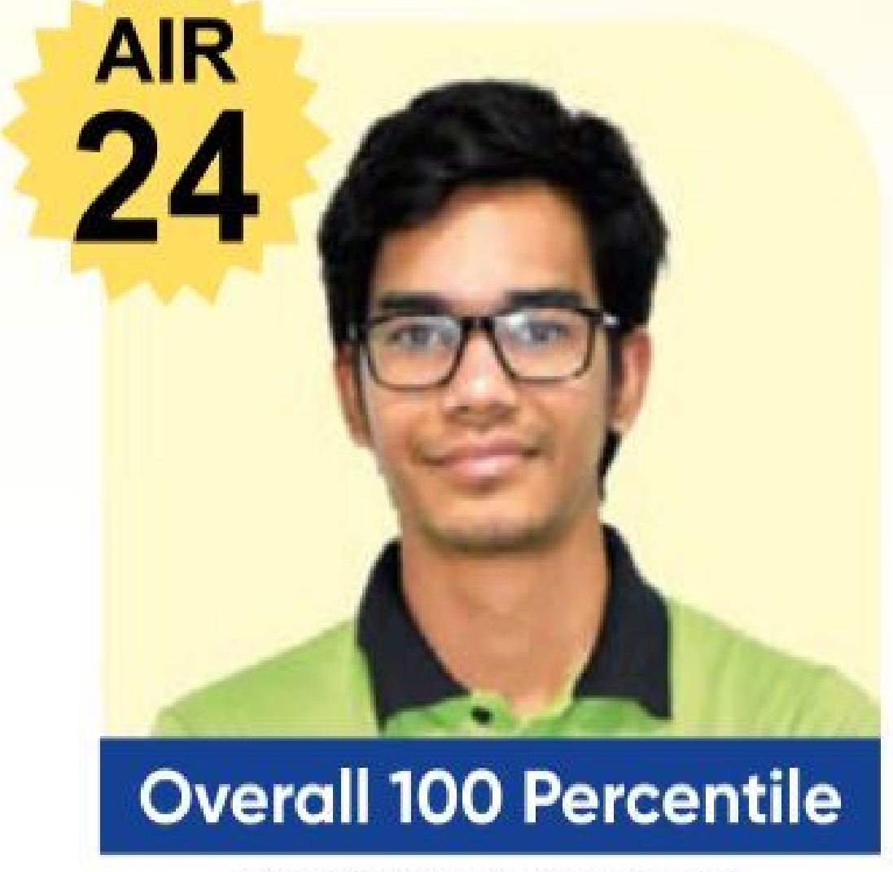

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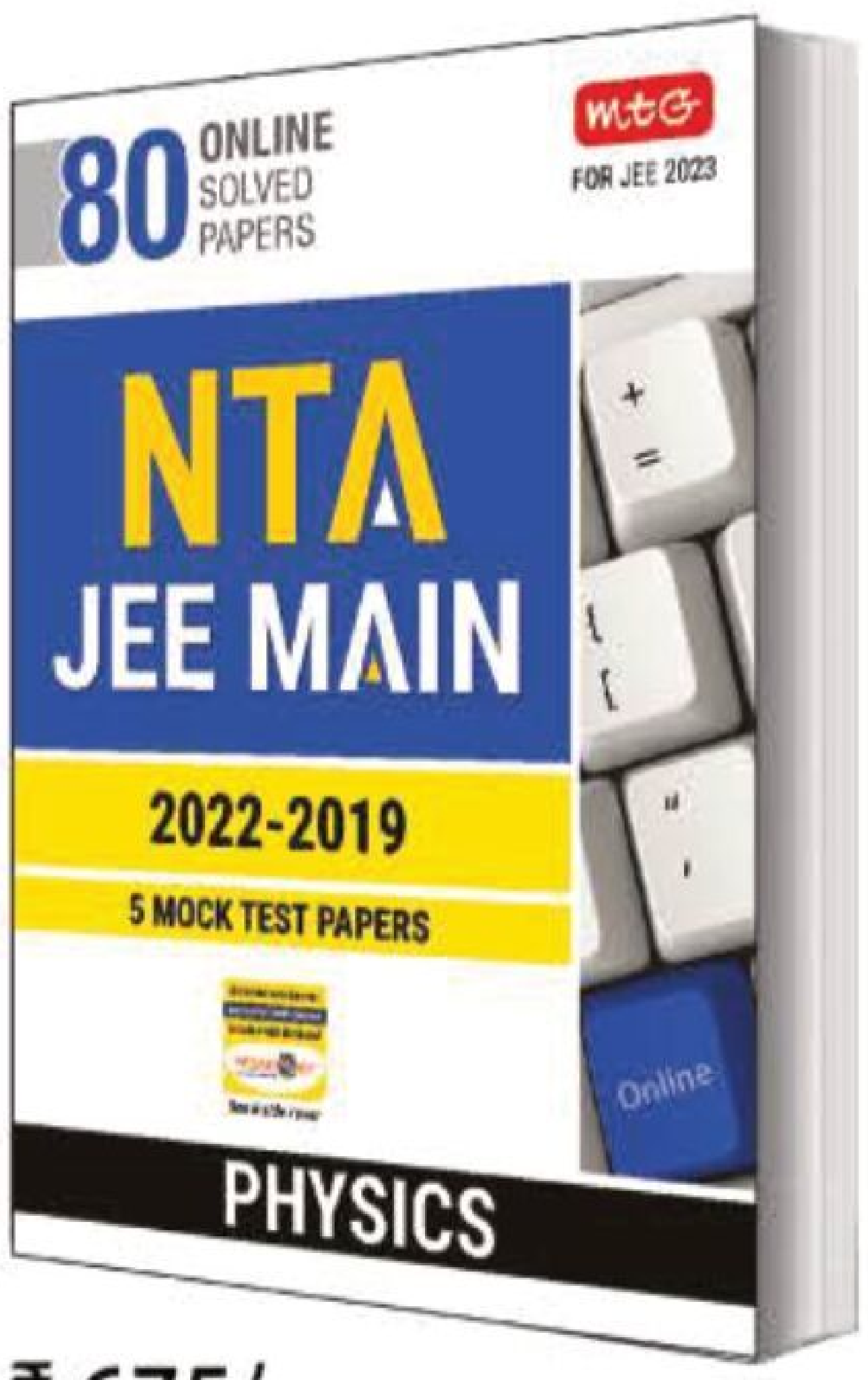
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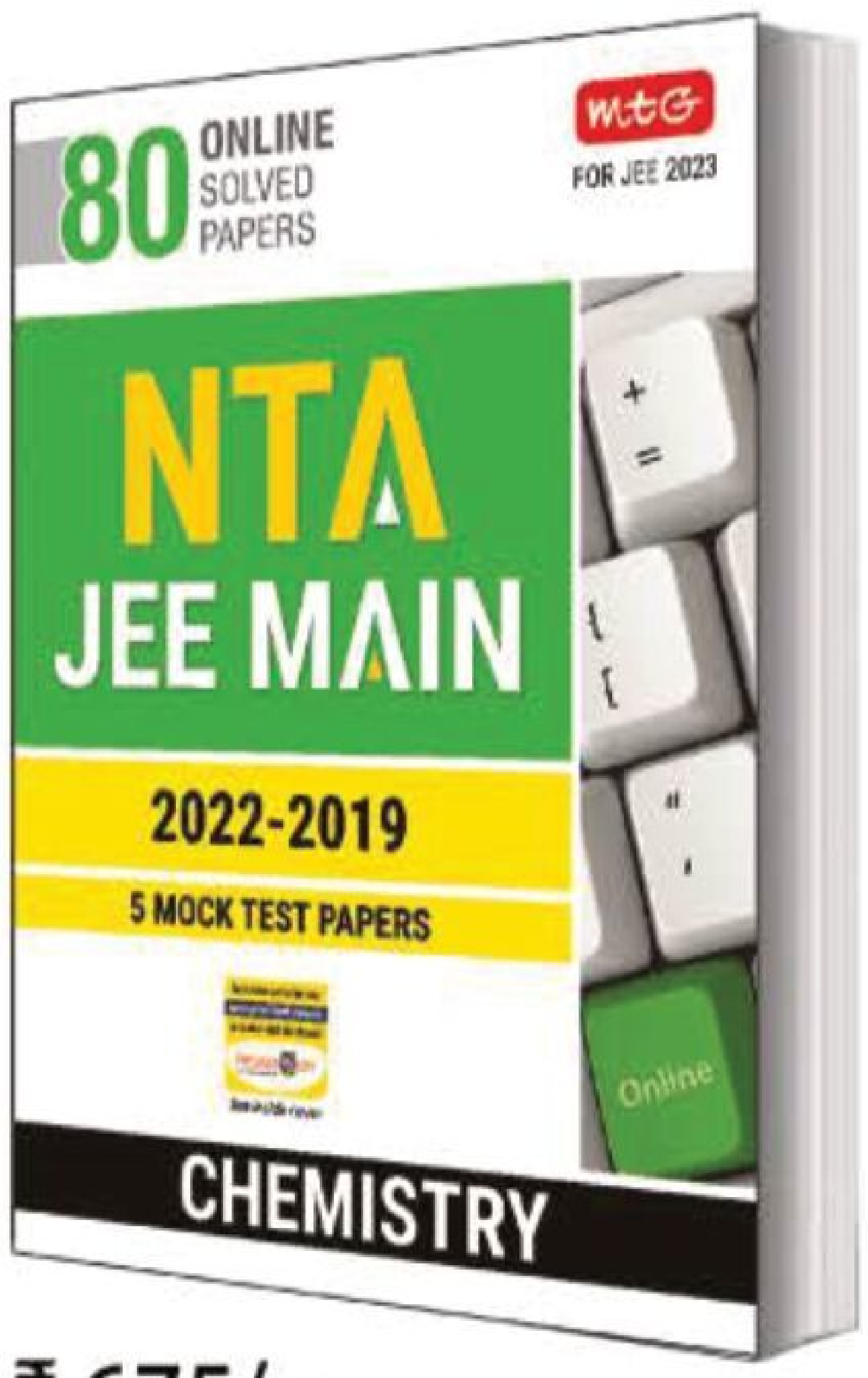
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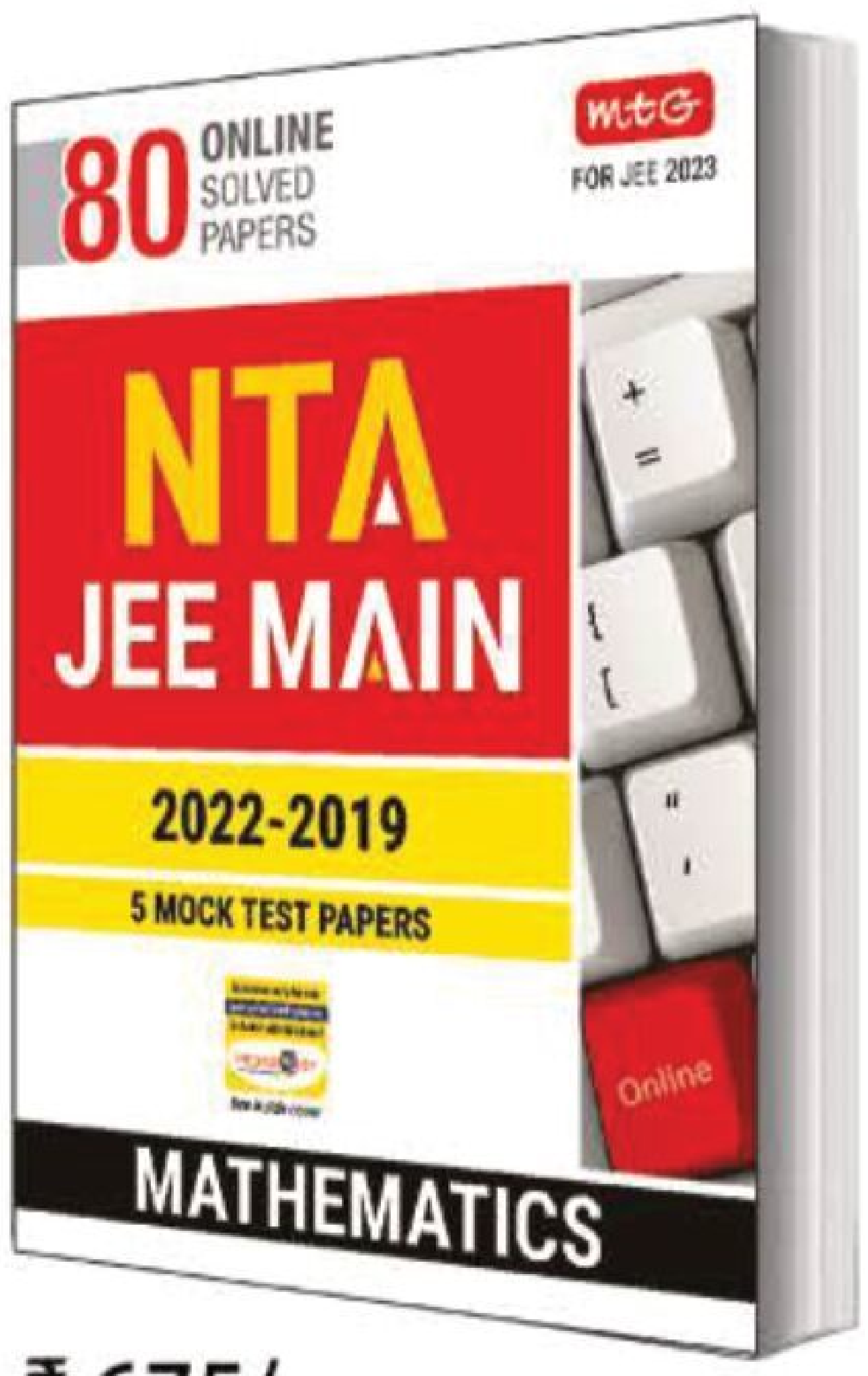
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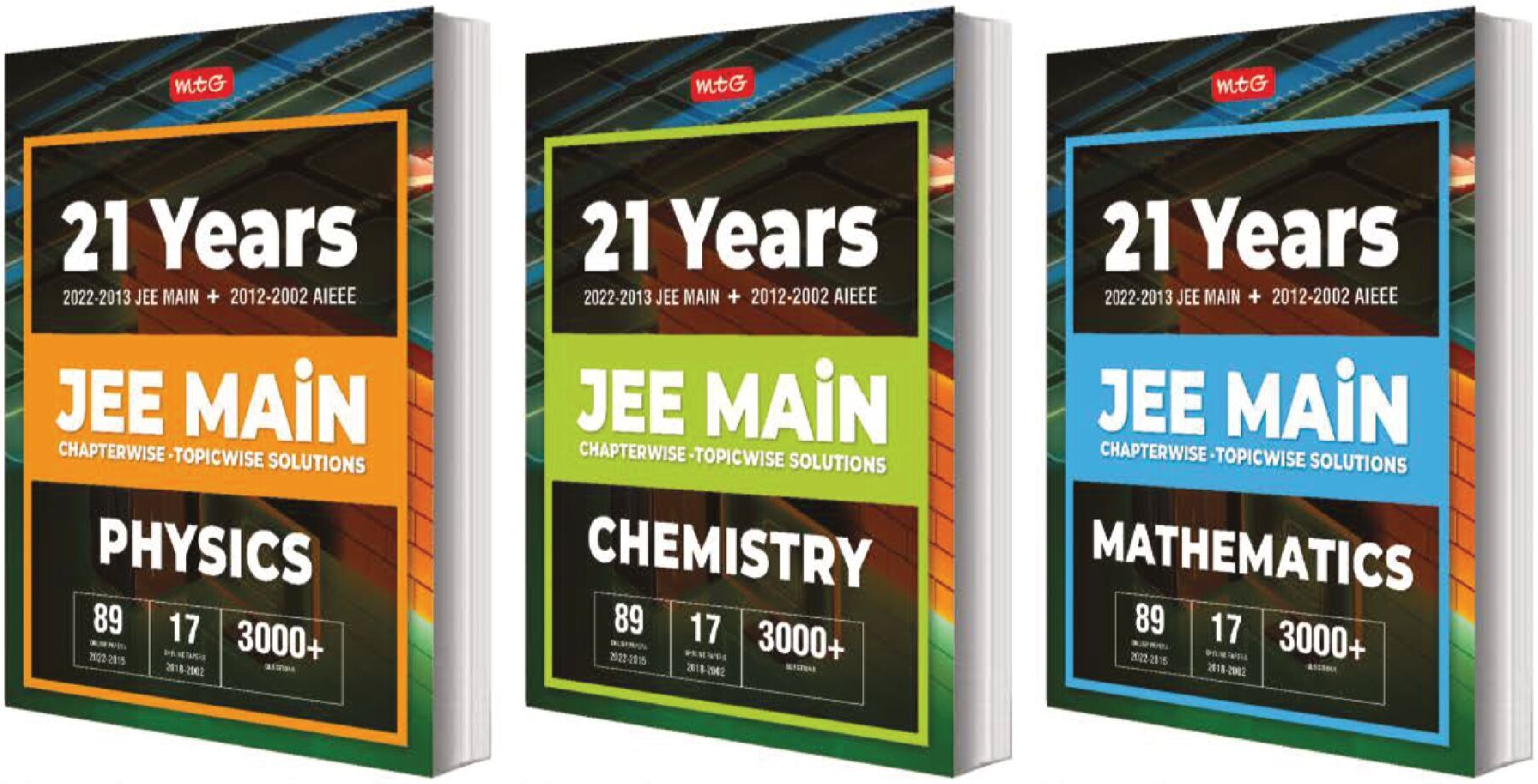
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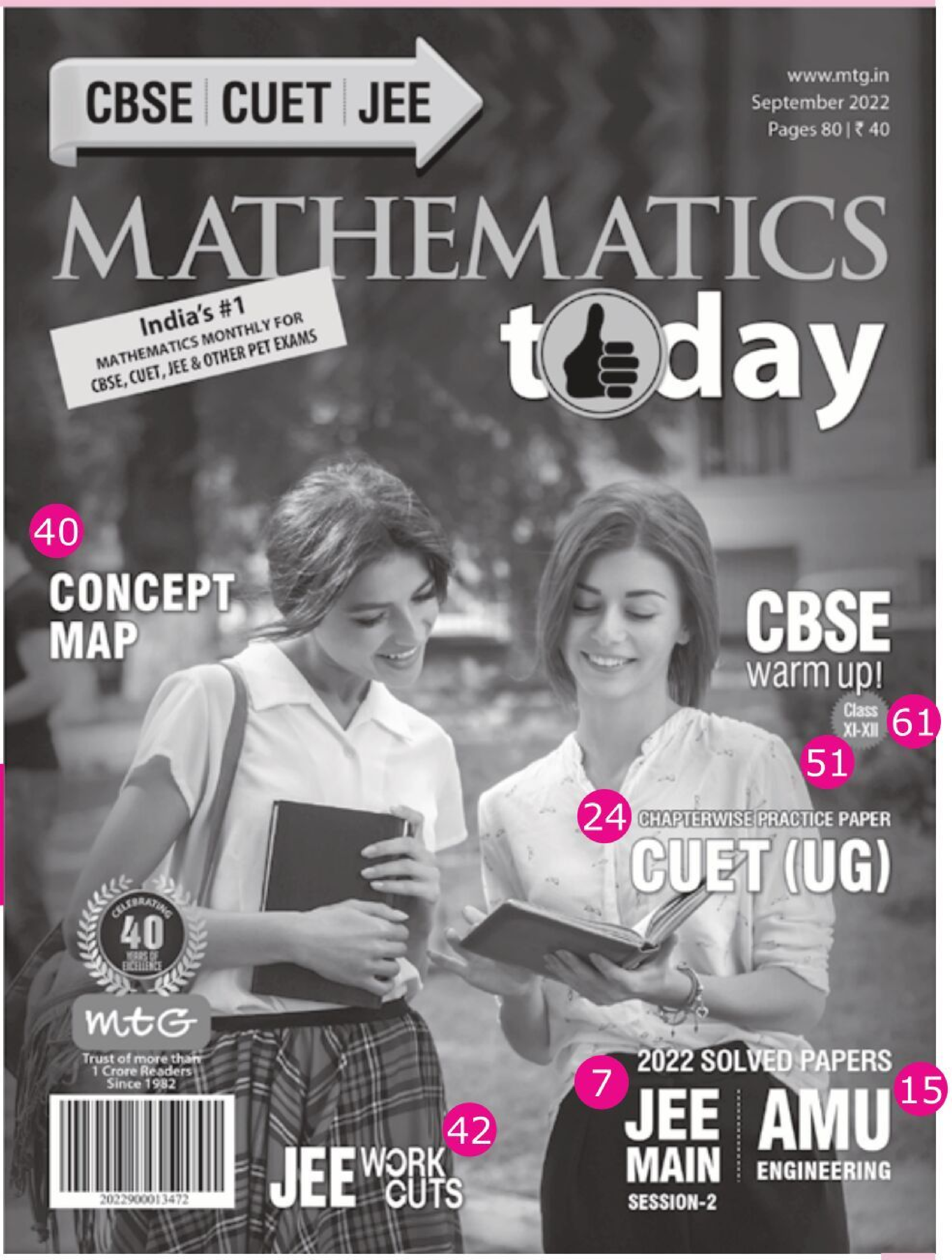
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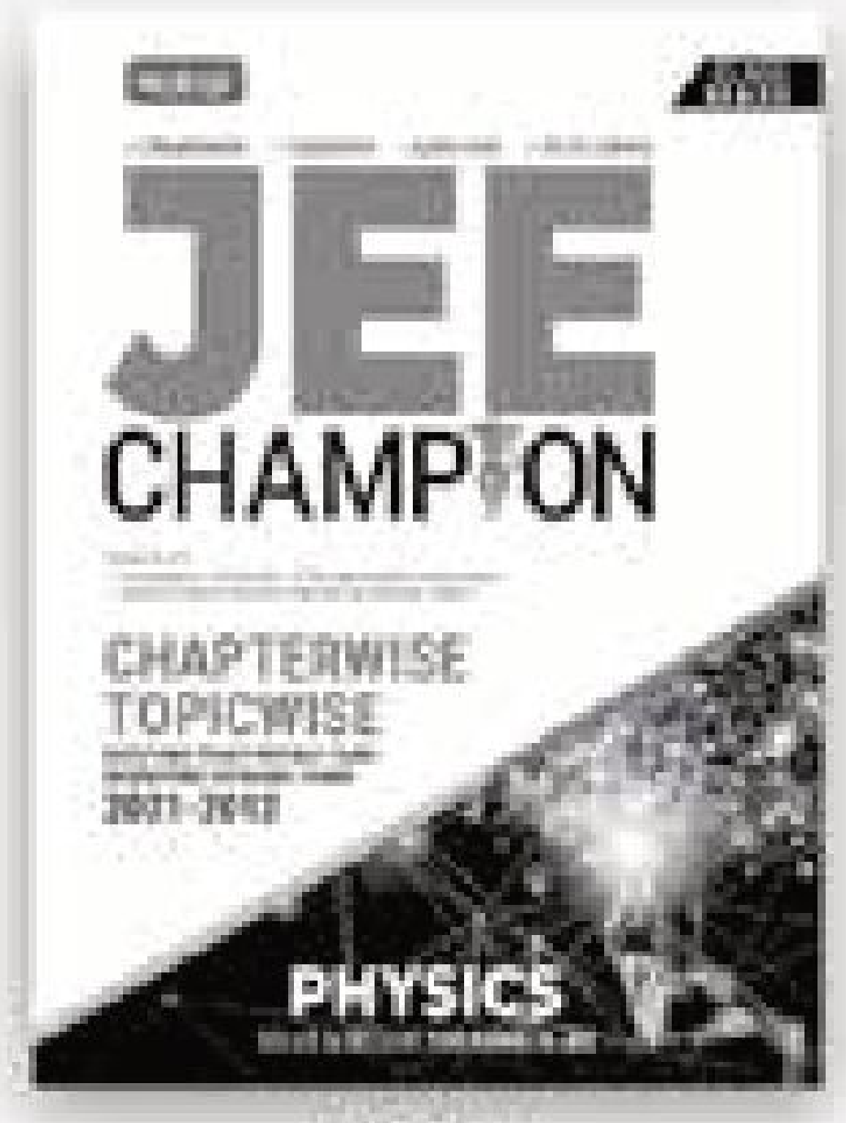
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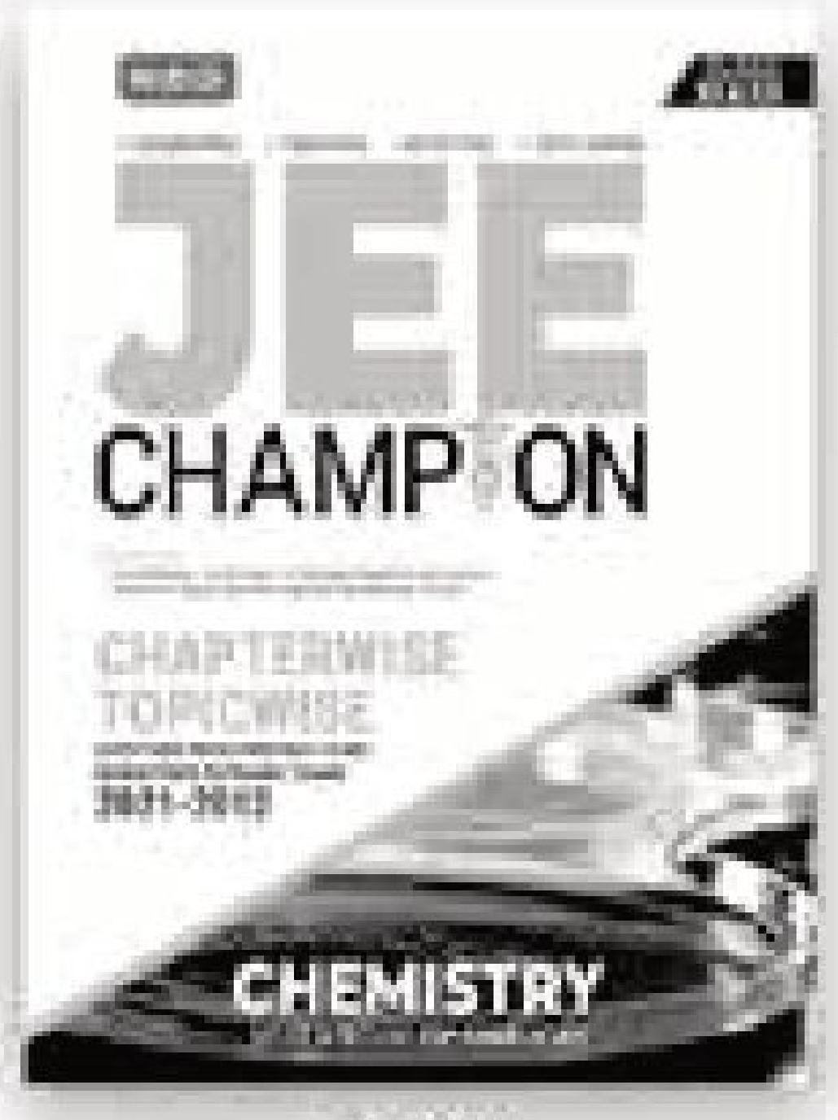
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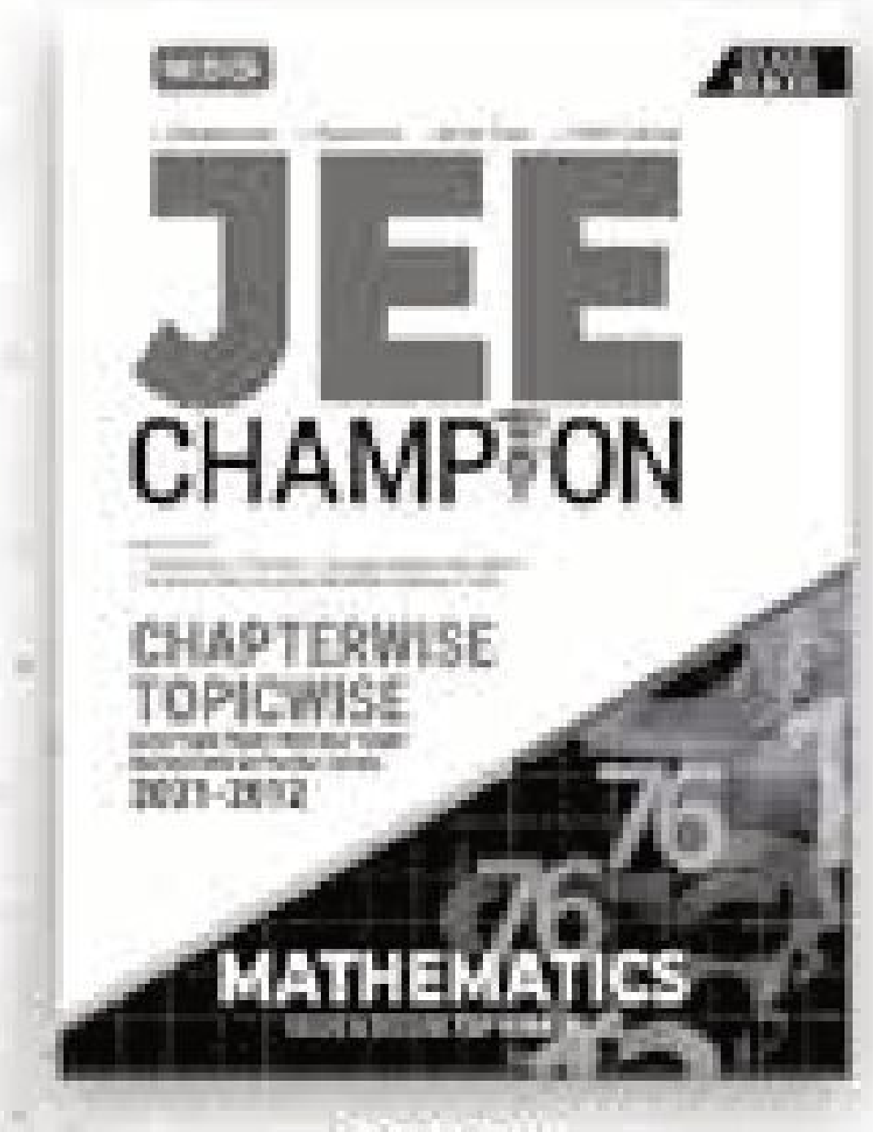
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Held on
25th July, Morning Shift

JEE MAIN 2022

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The total number of functions, $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to

(a) 60 (b) 90 (c) 108 (d) 126

2. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to

(a) -4 (b) -1 (c) 1 (d) 4

3. For $n \in N$, let $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ and $T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}$. Then the number of elements in the set $\{n \in N : S_n \cap T_n = \phi\}$ is

(a) 0 (b) 2 (c) 3 (d) 4

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$3(\sin 3\theta)\,x - y + z = 2$

$3(\cos 2\theta)\,x + 4y + 3z = 3$

$6x + 7y + 7z = 9$ has no solution, is

(a) 6 (b) 7 (c) 8 (d) 9

5. If $\lim\limits_{n \to \infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$, then $8(\alpha + \beta)$ is equal to

(a) 4 (b) -8 (c) -4 (d) 8

6. If the absolute maximum value of the function $f(x) = (x^2 - 2x + 7)\, e^{(4x^3 - 12x^2 - 180x + 31)}$ in the interval $[-3, 0]$ is $f(\alpha)$, then

(a) $\alpha = 0$ (b) $\alpha = -3$
(c) $\alpha \in (-1, 0)$ (d) $\alpha \in (-3, -1]$

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point $P(-2, 0)$ and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of $y(x)$ is

(a) $\frac{27}{4}$ (b) $\frac{29}{4}$ (c) $\frac{37}{4}$ (d) $\frac{9}{2}$

8. The area of the region given by $A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$ is

(a) $\frac{31}{8}$ (b) $\frac{17}{6}$ (c) $\frac{19}{6}$ (d) $\frac{27}{8}$

9. For any real number x, let $[x]$ denote the largest integer less than equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - [x], \text{ if } [x] \text{ is odd} \\ 1 + [x] - x, \text{ if } [x] \text{ is even} \end{cases}$$

. Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

(a) 4 (b) 2 (c) 1 (d) 0

10. The slope of the tangent to a curve $C : y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to

(a) $\frac{3 + \sqrt{2}}{3 - \sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}\left(\frac{3 + \sqrt{2}}{3 - \sqrt{2}}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right)$ (d) $\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$

11. The general solution of the differential equation $(x - y^2)\,dx + y(5x + y^2)dy = 0$ is

(a) $(y^2 + x)^4 = C|(y^2 + 2x)^3|$
(b) $(y^2 + 2x)^4 = C|(y^2 + x)^3|$
(c) $|(y^2 + x)^3| = C(2y^2 + x)^4$
(d) $|(y^2 + 2x)^3| = C(2y^2 + x)^4$

12. A line, with the slope greater than one, passes through the point $A(4, 3)$ and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line

(a) $2x + y = 9$ (b) $3x - 2y = 7$
(c) $x + 2y = 6$ (d) $2x - 3y = 3$

13. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is

(a) $\frac{32\sqrt{2}}{3}$ (b) $\frac{40\sqrt{2}}{3}$ (c) $\frac{64}{3}$ (d) $\frac{32}{3}$

14. Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point $(2, -5, 11)$, then d^2 is equal to

(a) $\frac{147}{2}$ (b) 96 (c) $\frac{32}{3}$ (d) 54

15. Let ABC be a triangle such that

$\overrightarrow{BC} = \vec{a}, \overrightarrow{CA} = \vec{b}, \overrightarrow{AB} = \vec{c},$

$|\vec{a}| = 6\sqrt{2}, |\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12.$

Consider the statements

$(\text{S1}): |(\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b})| - |\vec{c}| = 6(2\sqrt{2} - 1)$

$(\text{S2}): \angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Then

(a) both (S1) and (S2) are true
(b) only (S1) is true
(c) only (S2) is true
(d) both (S1) and (S2) are false

16. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is

(a) $\frac{33}{2^{32}}$ (b) $\frac{33}{2^{29}}$ (c) $\frac{33}{2^{28}}$ (d) $\frac{33}{2^{27}}$

17. If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in \mathbf{R}$, is

(a) $\frac{17}{36}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{19}{36}$

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is

(a) 4 (b) 6 (c) 8 (d) 12

19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that $QR = 15$ m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is

(a) $5(2\sqrt{3}+3)$ m (b) $5(\sqrt{3}+3)$ m
(c) $10(\sqrt{3}+1)$ m (d) $10(2\sqrt{3}+1)$ m

20. Which of the following statements is a tautology?

(a) $((\sim p) \vee q) \Rightarrow p$ (b) $p \Rightarrow ((\sim p) \vee q)$
(c) $((\sim p) \vee q) \Rightarrow q$ (d) $q \Rightarrow ((\sim p) \vee q)$

SECTION-B (NUMERICAL TYPE QUESTIONS)

Attempt any 5 questions out of 10.

21. Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}\,i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n + (\omega B)^n = A + B\}$ is equal to _______.

22. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is _______.

23. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \ge 0$, is K, then $8K$ is equal to _______.

24. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _______.

25. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \ge 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to _______.

26. Let $f(x) = \begin{cases} |4x^2 - 8x + 5|, \text{ if } 8x^2 - 6x + 1 \ge 0 \\ [4x^2 - 8x + 5], \text{ if } 8x^2 - 6x + 1 < 0 \end{cases}$, where

$[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in **R** where f is not differentiable is _______.

27. If $\lim\limits_{n\to\infty} \dfrac{(n+1)^{k-1}}{n^{k+1}}\,[(nk+1)+(nk+2)+\ldots+(nk+n)]$

$$= 33\cdot\lim_{n\to\infty}\frac{1}{n^{k+1}}\cdot\left[1^k+2^k+3^k+\ldots+n^k\right],$$

then the integral value of k is equal to _______.

28. Let the equation of two diameters of a circle $x^2+y^2-2x+2fy+1=0$ be $2px-y=1$ and $2x+py=4p$. Then the slope $m\in(0,\infty)$ of the tangent to the hyperbola $3x^2-y^2=3$ passing through the centre of the circle is equal to _______.

29. The sum of diameters of the circles that touch (i) the parabola $75x^2=64(5y-3)$ at the point $\left(\frac{8}{5},\frac{6}{5}\right)$ and (ii) the y-axis, is equal to _______.

30. The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane $P:\ ax-y-z=0$, $(a>0)$. If the image of the point $(1, 1, -5)$ in the plane P is (α,β,γ), then $\alpha+\beta-\gamma$ is equal to _______.

SOLUTIONS

1. (b): We have $f: \{1, 2, 3, 4\}\to\{1, 2, 3, 4, 5, 6\}$

$f(1)+f(2)=f(3)$

$f(3)$	$f(1)$	$f(2)$
2	1	1
3	1	2
	2	1
4	2	2
	1	3
	3	1
5	1	4
	2	3
	3	2
	4	1
6	1	5
	2	4
	3	3
	4	2
	5	1
Total		**15**

Also, $f(4)$ have 6 choices.

$\therefore$ Total ways $= 15\times 6 = 90$

2. (b): We have $x^4+x^3+x^2+x+1=0$.

α,β,γ and δ are the roots of given equation.

Since all these roots are roots of the equation x^5-1 except 1.

$\therefore\ \alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}=\alpha+\beta+\gamma+\delta=-1.$

3. (None of the options is correct) : We have,

$S_n=\left\{z\in C: |z-3+2i|=\frac{n}{4}\right\}$, which represents circle with centre $C_1(3, -2)$ and radius $r_1=\frac{n}{4}$

$T_n=\left\{z\in C: |z-2+3i|=\frac{1}{n}\right\}$, which represents circle with centre $C_2(2, -3)$ and radius $r_2=\frac{1}{n}$.

Also, $C_1C_2>r_1+r_2$ or $C_1C_2<r_1-r_2$

For $C_1C_2>r_1+r_2$

$\Rightarrow\ \sqrt{1+1}>\frac{n}{4}+\frac{1}{n}\ \Rightarrow\ \sqrt{2}>\frac{n}{4}+\frac{1}{n}\ \Rightarrow n=1, 2, 3, 4.$

And for $C_1C_2<r_1-r_2$

$\Rightarrow\ \sqrt{2}<\frac{n}{4}-\frac{1}{n}\ \Rightarrow n\geq 7$

$\therefore$ There can be infinite number of points.

Hence, none of the options is correct.

4. (b): For $\theta\in(0, 4\pi)$, the system of linear equations are $3(\sin 3\theta)\,x-y+z=2$; $3(\cos 2\theta)\,x+4y+3z=3$ and $6x+7y+7z=9$.

For no solution, $|A|=0$.

$$\begin{vmatrix} 3\sin 3\theta & -1 & 1\\ 3\cos 2\theta & 4 & 3\\ 6 & 7 & 7\end{vmatrix}=0$$

$\Rightarrow\ 3\sin 3\theta\,(28-21)+1(21\cos 2\theta-18)+1\,(21\cos 2\theta-24)=0$

$\Rightarrow\ \sin 3\theta+2\cos 2\theta-2=0$

$\Rightarrow\ -\sin\theta(4\sin^2\theta+4\sin\theta-3)=0$

$\Rightarrow\ \sin\theta=0$ or $4\sin^2\theta+4\sin\theta-3=0$

Now, $\sin\theta=0\Rightarrow\theta=\pi, 2\pi, 3\pi$

or $4\sin^2\theta+4\sin\theta-3=0\ \Rightarrow\ \sin\theta=\frac{1}{2}$

$\therefore\ \theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$

$\therefore$ Total number of possible values of θ is 7.

5. (c) : We have given that

$$\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$$

$$\Rightarrow \lim_{n\to\infty}\left[n\left(1-\frac{1+n}{n^2}\right)^{1/2}+n\alpha+\beta\right]=0$$

$$\Rightarrow \lim_{n\to\infty}\left[n\left\{1-\frac{1}{2}\left(\frac{1+n}{n^2}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{\lfloor\underline{2}}\cdot\left(\frac{1+n}{n^2}\right)^2+\ldots\right\}+n\alpha+\beta\right]=0$$

$$\Rightarrow \lim_{n\to\infty}\left[n-\frac{1}{2}\left(\frac{1+n}{n}\right)+\frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2}\cdot\frac{(1+n)^2}{n^3}+\ldots+n\alpha+\beta\right]=0$$

$$\Rightarrow \lim_{n\to\infty}\left[n-\frac{1}{2}+\ldots+n\alpha+\beta\right]=0$$

$\therefore\ \alpha=-1,\ \beta=\frac{1}{2}.$

6. (b): We have given the function

$f(x)=(x^2-2x+7)\,e^{(4x^3-12x^2-180x+31)}$

$f'(x)=(x^2-2x+7)\,e^{(4x^3-12x^2-180x+31)}\cdot$

$\quad(12x^2-24x-180)+e^{(4x^3-12x^2-180x+31)}\cdot(2x-2)$

$=e^{(4x^3-12x^2-180x+31)}\left[(x^2-2x+7)(12x^2-24x-180)+2x-2\right]$

$=e^{(4x^3-12x^2-180x+31)}$

$\quad[12(x^2-2x+7)(x+3)(x-5)+2(x-1)]$

$\therefore$ $f'(x)$ is negative in $[-3, 0]$.

$\therefore$ The absolute maximum value of the function is at $x=-3$.

7. (a): Given curve is $y(x)=ax^3+bx^2+cx+5$

Curve passes through the point $P(-2, 0)$.

$\therefore\ -8a+4b-2c+5=0$ ...(i)

$y'(x)=3ax^2+2bx+c$ ...(ii)

Also, curve touches the x-axis at point $(-2, 0)$.

$\therefore\ 12a-4b+c=0$ ...(iii)

When $x=0$, $y'=3$ $\therefore$ From (ii), $c=3$.

Put $c=3$ in (i) and (iii), we get $8a-4b+1=0$ and $12a-4b+3=0$

After solving these equations, we get

$a=-\frac{1}{2},\ b=-\frac{3}{4}$

$\therefore$ Equation of curve is $y(x)=\frac{-1}{2}x^3-\frac{3}{4}x^2+3x+5$

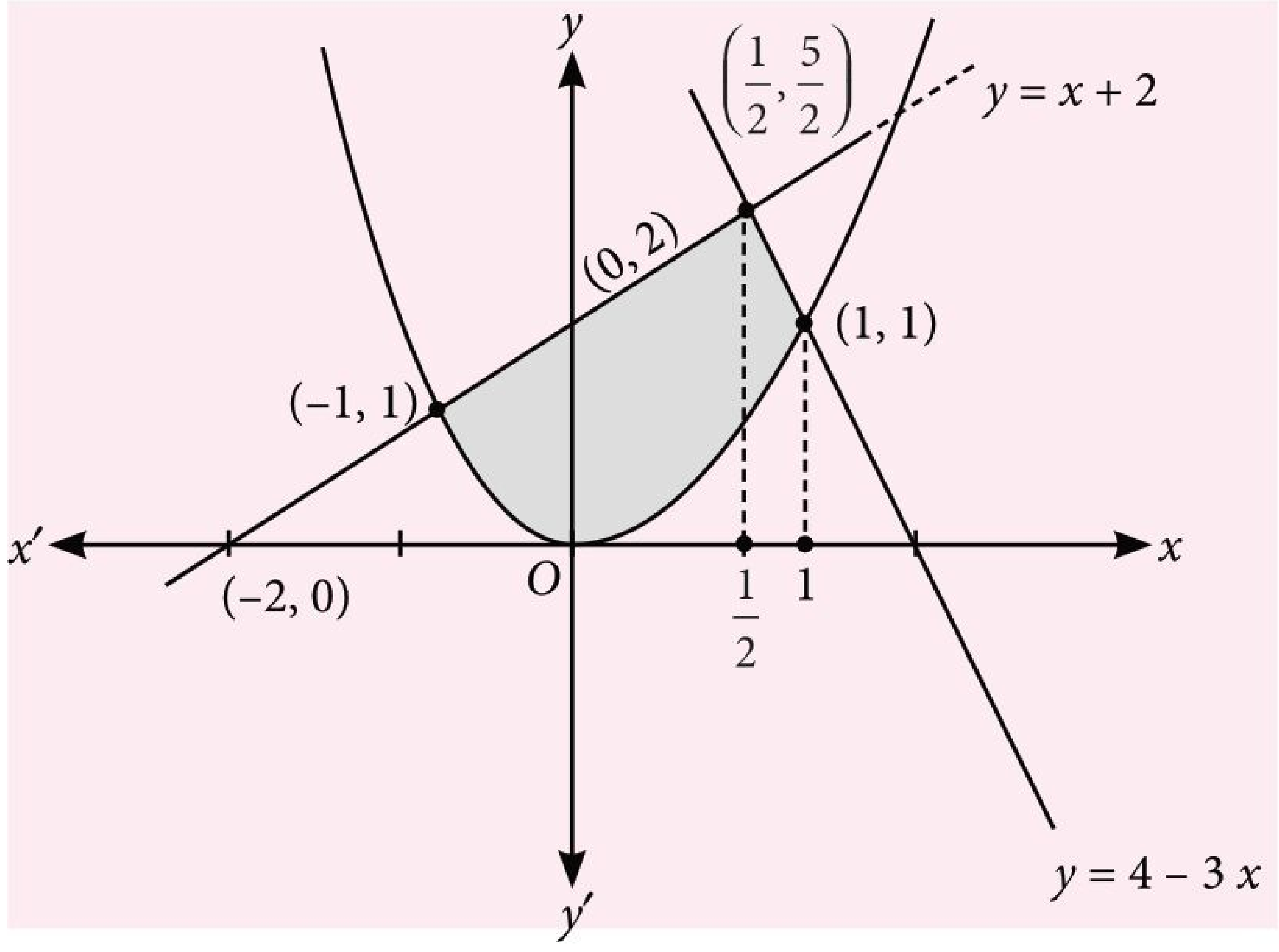
$y'(x)=\frac{-3x^2}{2}-\frac{3}{2}x+3$

For maxima or minima, $y'(x)=0 \Rightarrow x=1, -2$

$\therefore$ At $x=1$, $y(x)$ has local maxima.

$\therefore\ y(1)=\frac{27}{4}$

8. (b): We have, $A=\{(x, y): x^2\le y\le \min\{x+2, 4-3x\}\}$



$$\text{Required area}=\int_{-1}^{1/2}(x+2-x^2)\,dx+\int_{1/2}^{1}(4-3x-x^2)\,dx$$

$$=\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{1/2}+\left[4x-\frac{3x^2}{2}-\frac{x^3}{3}\right]_{1/2}^{1}=\frac{17}{6}$$

9. (a): We have, $f(x)=\begin{cases}\{x\}, & 2n-1\le x<2n\\ 1-\{x\}, & 2n\le x<2n+1\end{cases}$

Clearly, $f(x)$ is periodic function having period 2.

So, $f(x)\cos\pi x$ is also periodic with period 2.

Now, $\frac{\pi^2}{10}\int_{-10}^{10}f(x)\cos\pi x\,dx$

$$=\pi^2\int_0^2 f(x)\cos\pi x\,dx\ =-2\pi^2\int_0^1 x\cos\pi x\,dx$$

$$=-2\pi^2\left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^1=-2\pi^2\left[-\frac{1}{\pi^2}-\frac{1}{\pi^2}\right]=4$$

10. (b): Given that $\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$

$$\frac{dy}{dx}=e^{2x}-\frac{6e^x}{2e^{2x}+9};\ dy=\left(e^{2x}-\frac{6e^x}{2e^{2x}+9}\right)dx$$

On integrating both sides, we get

$$y=\frac{e^{2x}}{2}-\int\frac{6e^x}{2e^{2x}+9}\,dx$$

$$y=\frac{e^{2x}}{2}-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,e^x}{3}\right)+c$$

Since, given curve passes through $\left(0, \frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$.

$$\therefore\ \frac{1}{2}+\frac{\pi}{2\sqrt{2}}=\frac{1}{2}-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)+c$$

$$c=\frac{\pi}{2\sqrt{2}}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

$$c=\sqrt{2}\left[\frac{\pi}{4}+\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)\right]$$

Also, the curve passes through $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$.

$$\therefore\ \frac{1}{2}e^{2\alpha}=\frac{e^{2\alpha}}{2}-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,e^{\alpha}}{3}\right)+\sqrt{2}\left[\frac{\pi}{4}+\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)\right]$$

$$\Rightarrow\ e^{\alpha}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right).$$

11. (a) : $(x-y^2)\,dx+y\,(5x+y^2)\,dy=0$

$$\Rightarrow\ \frac{dy}{dx}=\frac{y^2-x}{y(5x+y^2)}$$

Put $y^2=t \Rightarrow 2y\frac{dy}{dx}=\frac{dt}{dx}$; $\frac{2(t-x)}{5x+t}=\frac{dt}{dx}$

Put $t=vx$

$$\Rightarrow\ \frac{dt}{dx}=v+x\frac{dv}{dx}$$

$$\therefore\ v+x\frac{dv}{dx}=\frac{2(vx-x)}{5x+vx}=\frac{2(v-1)}{5+v}$$

$$\Rightarrow\ \frac{(5+v)}{(v^2+3v+2)}dv=-\frac{dx}{x}$$

$$\Rightarrow\ \int\frac{5+v}{(v+1)(v+2)}dv=-\log x+\log c$$

$$\Rightarrow\ \int\left(\frac{4}{v+1}-\frac{3}{v+2}\right)dx=-\log x+\log c$$

$\Rightarrow$ $4\log(v+1)-3\log(v+2)=\log c-\log x$

$$\Rightarrow\ \frac{(v+1)^4}{(v+2)^3}=\frac{c}{x}$$

$$\Rightarrow\ x\left(\frac{y^2}{x}+1\right)^4=c\left(\frac{y^2}{x}+2\right)^3 \Rightarrow (y^2+x)^4=c(y^2+2x)^3$$

12. (c) : Let $B(\alpha, \alpha-2)$, $A\equiv(4, 3)$

$$AB=\frac{\sqrt{29}}{3}$$

$$\therefore\ \sqrt{(\alpha-4)^2+(\alpha-2-3)^2}=\frac{\sqrt{29}}{3}\ \Rightarrow\ \alpha=\frac{17}{3},\frac{10}{3}$$

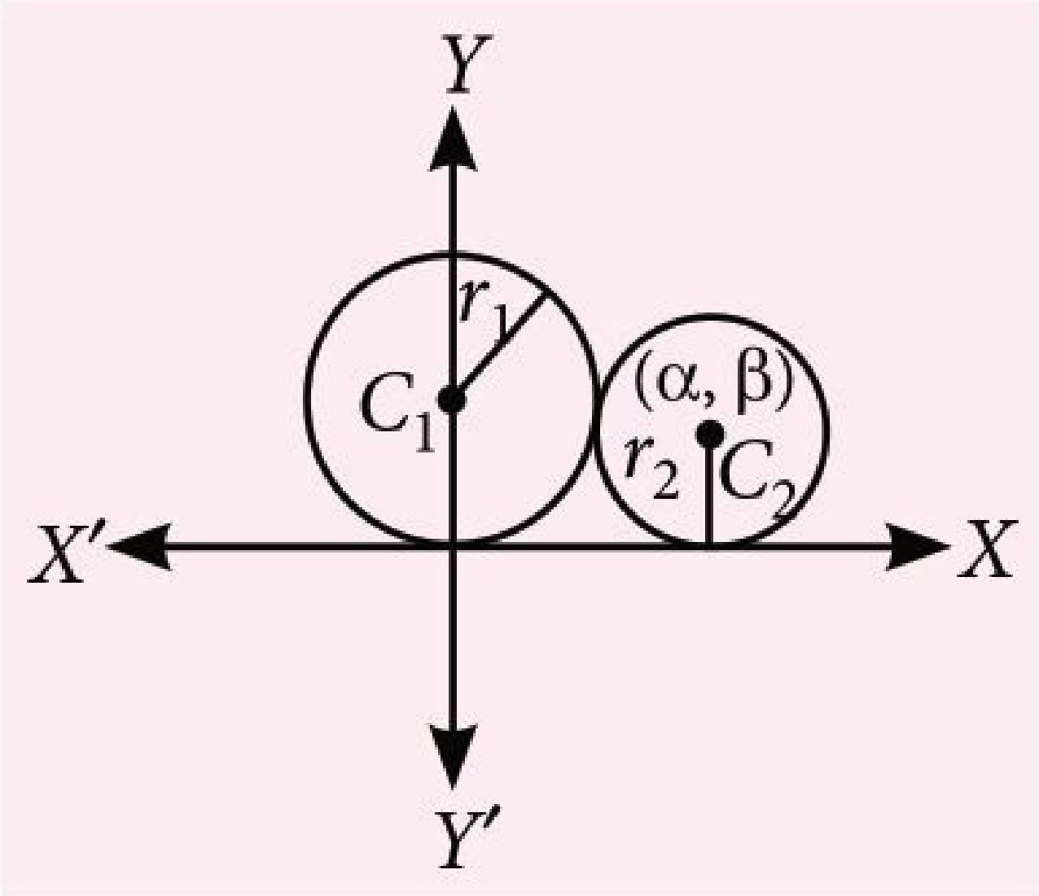
$$\therefore\ B\equiv\left(\frac{17}{3},\frac{11}{3}\right)\text{ or }\left(\frac{10}{3},\frac{4}{3}\right)$$

From the given option, the correct coordinates of B are $\left(\frac{10}{3},\frac{4}{3}\right)$, which lies on $x+2y=6$.

13. (c) : We have $C: x^2+(y-1)^2=1$

Centre is $C_1(0, 1)$ and radius = 1.

The another circle whose centre is $C_2(\alpha, \beta)$ and touches the x-axis. Therefore the radius will be β.



Also, $C_1C_2=r_1+r_2$

$$\Rightarrow\ \sqrt{(0-\alpha)^2+(1-\beta)^2}=\beta+1$$

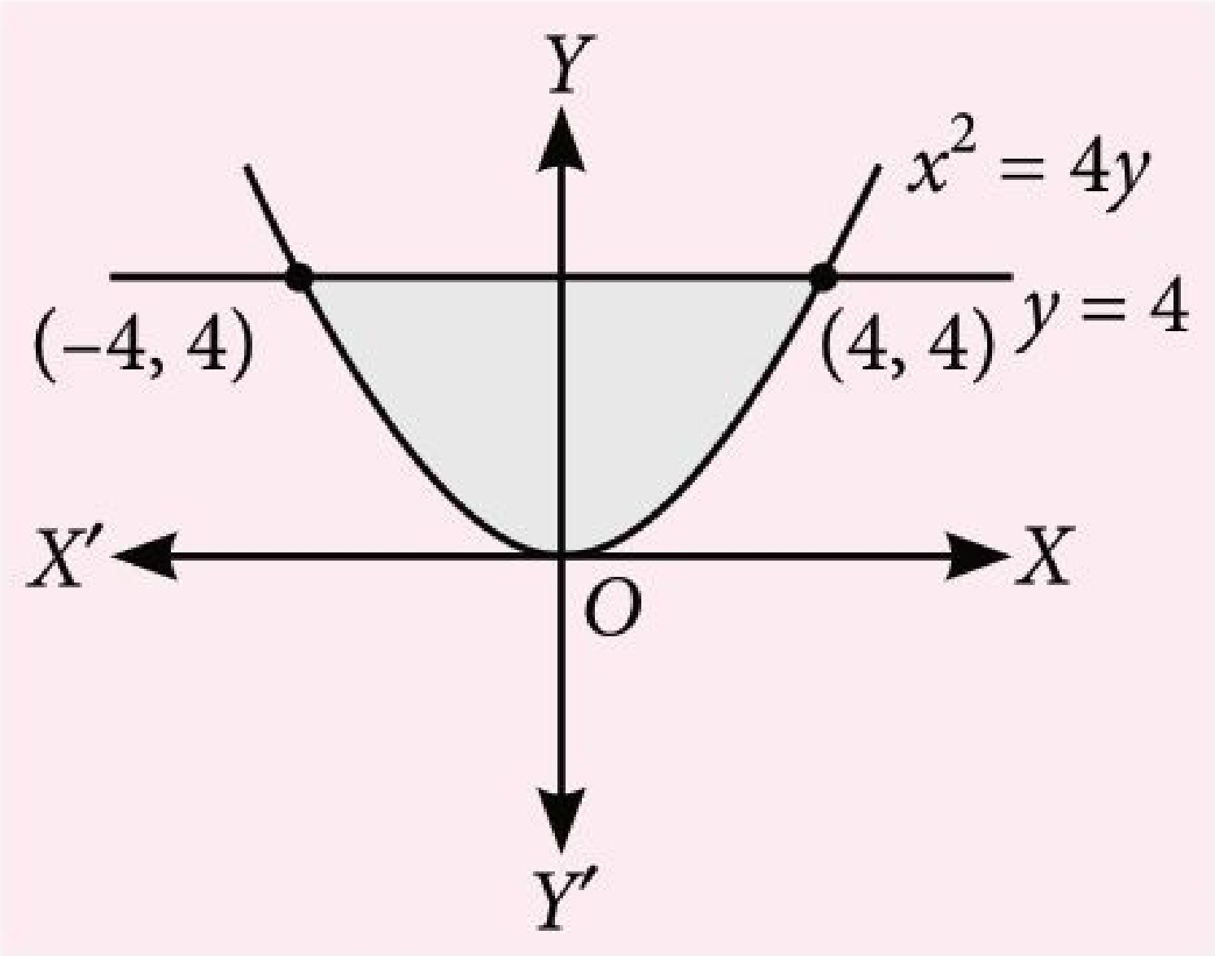
$\Rightarrow$ $\alpha^2+(1-\beta)^2=(\beta+1)^2$

$\Rightarrow$ $\alpha^2+1+\beta^2-2\beta$

$=\beta^2+1+2\beta$

$\Rightarrow$ $\alpha^2=4\beta$

$\therefore$ The locus of the centre is $x^2=4y$.



$$\therefore\ \text{Area}=2\int_0^4\left(4-\frac{x^2}{4}\right)dx$$

$$=2\left[4x-\frac{x^3}{12}\right]_0^4$$

$$=\frac{64}{3}\text{ sq. units.}$$

14. (c) : We have $L_1:\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$

$$L_2:\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$$

$$L_3:\frac{x}{3}=\frac{y}{7}=\frac{z}{8}$$

Let the equation of the plane P is

$a(x-3)+b(y+4)+c(z-7)=0$ ...(i)

The normal of plane P is $\perp^r$ to L_1

$\therefore$ $9a-b-5c=0$ ...(ii)

Normal of plane containing the line L_2 and L_3 is given by

$$\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 5\\ 3 & 7 & 8\end{vmatrix}=-11\hat{i}-\hat{j}+5\hat{k}$$

Clearly, $-11a-b+5c=0$ ...(iii)

Solving (ii) and (iii), we get $\frac{a}{1}=\frac{b}{-1}=\frac{c}{2}$

From (i), $(x-3)-(y+4)+2(z-7)=0$

$$\Rightarrow\ x-y+2z-21=0 \Rightarrow\ d=\frac{8}{\sqrt{6}}\ \Rightarrow\ d^2=\frac{32}{3}$$

15. (c) : We have $\overrightarrow{BC}=\vec{a},\ \overrightarrow{CA}=\vec{b},\ \overrightarrow{AB}=\vec{c}$

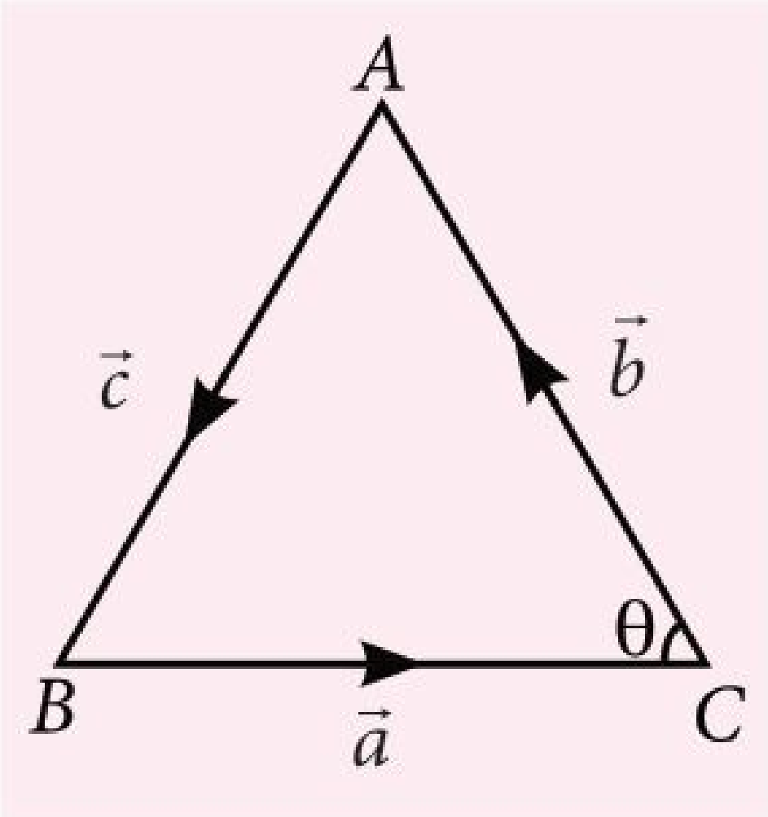
$|\vec{a}|=6\sqrt{2},\ |\vec{b}|=2\sqrt{3},\ \vec{b}\cdot\vec{c}=12$

For triangle ABC,

$\vec{a}+\vec{b}=-\vec{c}$

$\therefore\ \vec{a}+\vec{c}=-\vec{b}$

S1: $\left|(\vec{a}\times\vec{b})+(\vec{c}\times\vec{b})\right|-|\vec{c}|=6(2\sqrt{2}-1)$



Now, $\left|(\vec{a}\times\vec{b})+(\vec{c}\times\vec{b})\right|-|\vec{c}|=\left|(\vec{a}+\vec{c})\times\vec{b}\right|-|\vec{c}|$

$=|-\vec{b}\times\vec{b}|-|\vec{c}| = 0-6=-6$

S2 : $\angle ACB=\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

$\because\ \vec{a}+\vec{b}+\vec{c}=0\,;\ \left|\vec{a}+\vec{b}\right|^2=|-\vec{c}|^2$

$\therefore\ \ |\vec{a}|^2+|\vec{b}|^2-2|\vec{a}|\cdot|\vec{b}|\cos\theta=|\vec{c}|^2;\ \cos\theta=\sqrt{\frac{2}{3}}$

S2 is true.

16. (c) : Mean + variance = 24

(Mean)(variance) = 128.

$\because\ \ np+npq=24$ ...(i)

$np\cdot npq=128$ ...(ii)

Solving (i) and (ii), we get

$p=\frac{1}{2}, q=\frac{1}{2}$, $n=32$.

$\therefore\ \ P(X=1)+P(X=2)={}^{32}C_1\,pq^{31}+{}^{32}C_2\,p^2q^{30}=\frac{33}{2^{28}}$

17. (a) : We have given $x^2+\alpha x+\beta>0$,

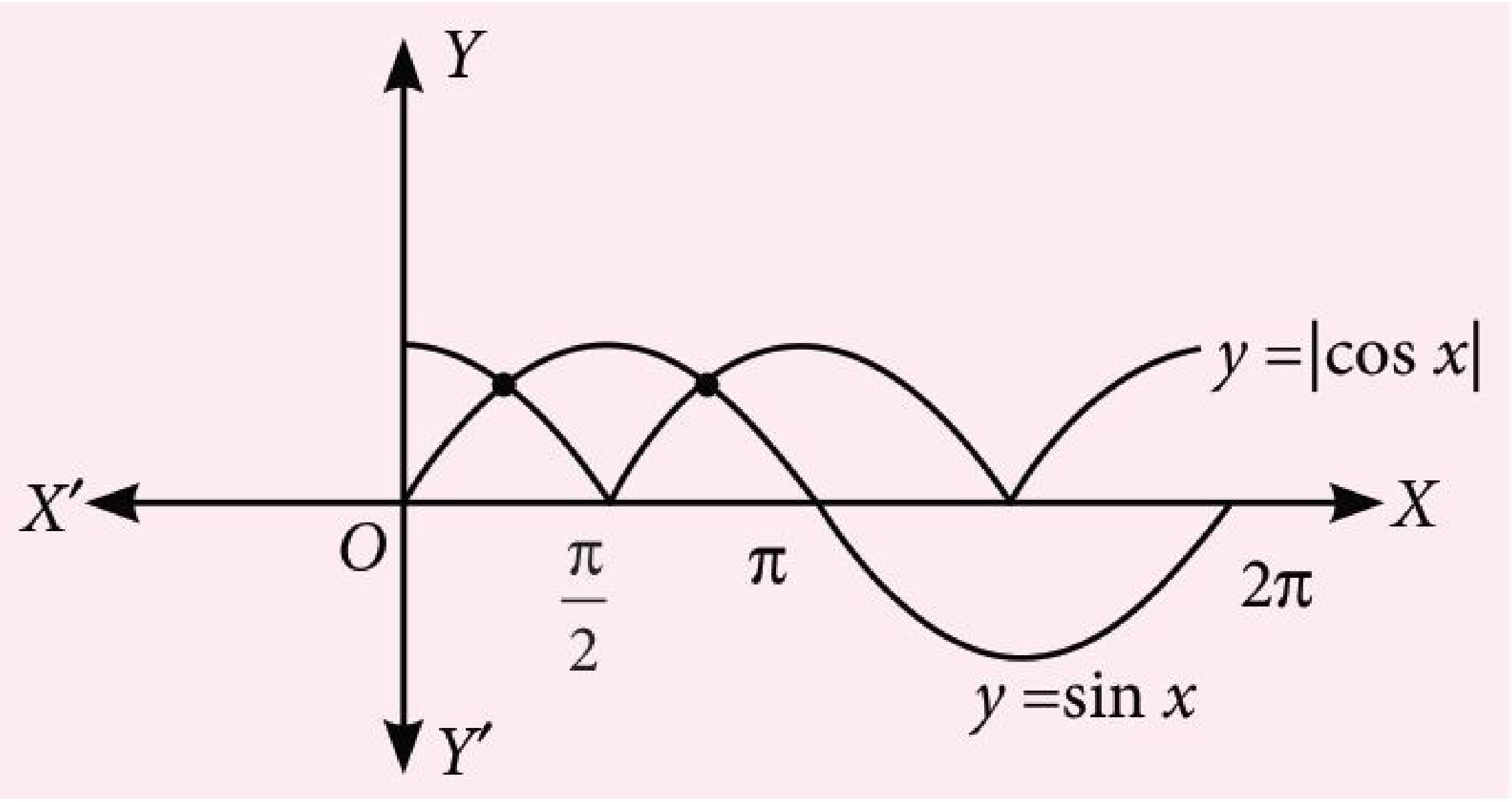
$\therefore\ \ D<0$

$\alpha^2-4\beta<0\Rightarrow\alpha^2<4\beta$.

$(\alpha,\beta)=\{(1,1),(1,2),\ldots,(1,6),(2,2),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,5),(4,6)\}$

$\therefore$ Required probability $=\frac{17}{36}$.

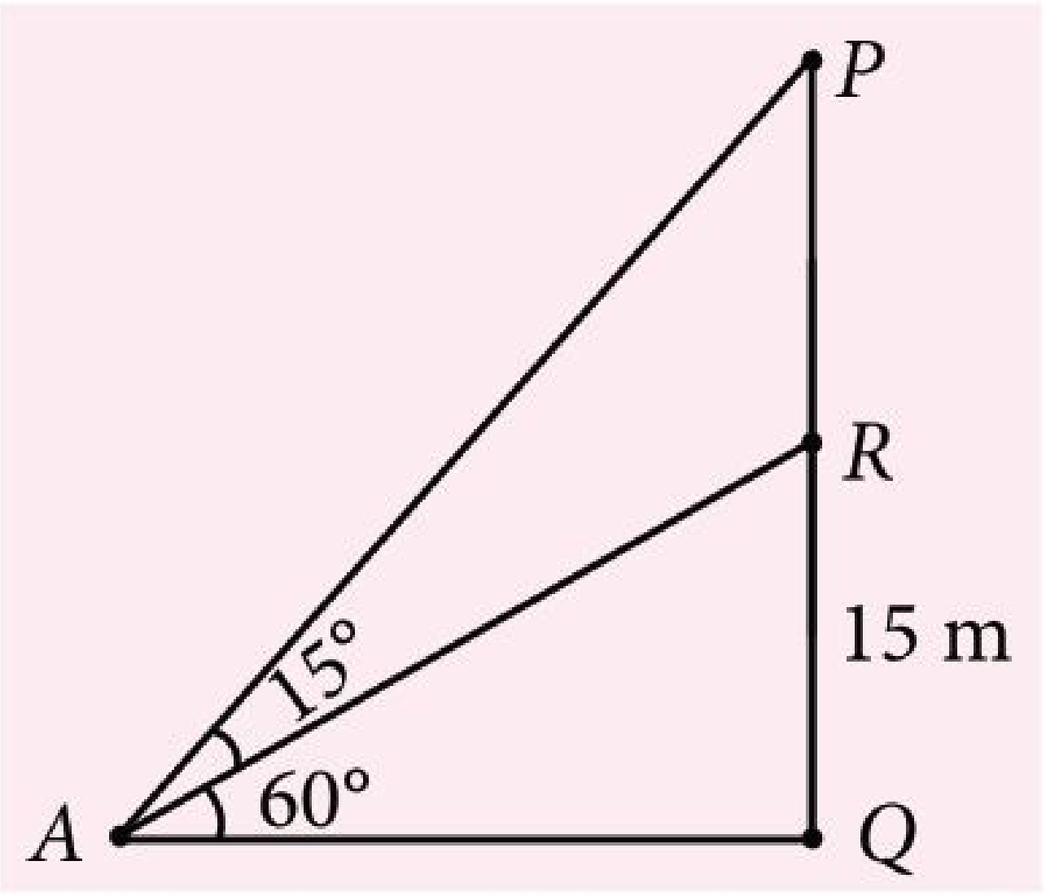
18. (c) : We have $|\cos x|=\sin x$, $x\in[-4\pi,4\pi]$



For interval $[0, 2\pi]$, there are 2 solutions.

$\therefore$ For interval $[-4\pi, 4\pi]$, there are 8 solutions.

19. (a) : Let $PR = x$ m



$\tan 60^\circ=\frac{15}{AQ}$ ...(i)

$\tan 75^\circ=\frac{x+15}{AQ}$ ...(ii)

On dividing (i) by (ii), we get

$\frac{\sqrt{3}}{2+\sqrt{3}}=\frac{15}{x+15}$

$\Rightarrow\ \ x+15=\frac{15\times(2+\sqrt{3})}{\sqrt{3}}\ \ \Rightarrow\ \ x=10\sqrt{3}$

$\therefore\ \ PQ=15+10\sqrt{3}\ \Rightarrow 5(3+2\sqrt{3})$

20. (d) : Consider option (d). The truth table is:

p	q	$\sim p$	$\sim q$	$\sim p\vee q$	$q\Rightarrow((\sim p)\vee q)$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	T	T

21. (17) : We have $A=\begin{bmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{bmatrix}$

$A^2=A\quad\therefore\ A^n=A$

$B=\begin{bmatrix}1 & -1 & -1\\ 1 & -1 & -1\\ 1 & -1 & -1\end{bmatrix}\Rightarrow B^2=-B$

$B^3=B(-B)=-B^2=B$

$\therefore\ \ B^5=B,\ \ B^{99}=B$

$\omega^3=1,\ \omega^{3k}=1$

$\therefore\ \ A^n+(\omega B)^n=A+B$

$\therefore\ \ n\in\{1,3,5,\ldots,99\}\cap\{3,6,9,\ldots,99\}$

$\therefore\ \ n=17$

mtG

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22. (1492) : A D I K M N

A -------------------- $\frac{6!}{2}$

D -------------------- $\frac{6!}{2}$

I -------------------- $\frac{6!}{2}$

K -------------------- $\frac{6!}{2}$

M A D ------------- $\frac{4!}{2}$

M A I -------------- $\frac{4!}{2}$

M A K ------------- $\frac{4!}{2}$

M A N D ----------- $\lfloor\underline{3}$

M A N I ------------ $\lfloor\underline{3}$

M A N K D --------- $\lfloor\underline{2}$

M A N K I D ------- $\lfloor\underline{1}$

M A N K I N D ---- 1

$\therefore$ Serial number of the word MANKIND = 1492.

23. (6006) : $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

The general term is

$$T_{r+1}={}^{15}C_r\left(t^2x^{\frac{1}{5}}\right)^{15-r}\cdot\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$$

$$={}^{15}C_r\,x^{\frac{15-r}{5}}\cdot(1-x)^{\frac{r}{10}}\cdot t^{30-2r-r}$$

For independent of t,

$30 - 2r - r = 0 \Rightarrow r = 10$

$\therefore\ T_{11} = {}^{15}C_{10}\cdot x\,(1-x)$

$\therefore\ T_{11}$ will be maximum when $x=\frac{1}{2}$

$\therefore\ K=\frac{3003}{4}\ \Rightarrow\ 8K = 6006.$

24. (38) : p and r are the roots of the equation, $x^2 - 8ax + 2a = 0$

$\therefore\ p + r = 8a,\ pr = 2a\ \Rightarrow\ \frac{1}{p}+\frac{1}{r}=4$ ...(i)

Also, q and s are the roots of $x^2 + 12bx + 6b = 0$

$\therefore\ q + s = -12b,\ qs = 6b\ \Rightarrow\ \frac{1}{q}+\frac{1}{s}=-2$ ...(ii)

Given that $\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}$ are in A.P.

$\therefore\ \frac{1}{q}-\frac{1}{p}=\frac{1}{r}-\frac{1}{q}\Rightarrow\frac{2}{q}=\frac{1}{r}+\frac{1}{p}=4$ [Using (i)]

$\Rightarrow\ q=\frac{1}{2}$ and $s=\frac{-1}{4}$

Clearly, $\frac{1}{r}-\frac{1}{q}=\frac{1}{s}-\frac{1}{r}\ \Rightarrow\ \frac{2}{r}=\frac{1}{q}+\frac{1}{s}$

$\Rightarrow\ \frac{2}{r}=-2\Rightarrow r=-1$ and $p=\frac{1}{5}$ [Using (ii)]

$\therefore\ \frac{1}{a}-\frac{1}{b}=\frac{2}{pr}-\frac{6}{qs}=38.$

25. (27560) : We have given $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$, $b_n = a_n + b_{n-1}$

$$\begin{array}{l|l} a_2=3 & b_2=4 \\ a_3=5 & b_3=9 \\ a_4=7 & b_4=16 \\ a_5=9 & \therefore\ b_n=n^2 \\ a_6=11 & \\ a_8=13 & \end{array}$$

$$\therefore\ \sum_{n=1}^{15}a_n\cdot b_n=\sum_{n=1}^{15}(2n-1)\,n^2=\sum_{n=1}^{15}(2n^3-n^2)$$

$$=2\cdot\frac{n^2(n+1)^2}{4}-\frac{n(n+1)(2n+1)}{6}$$

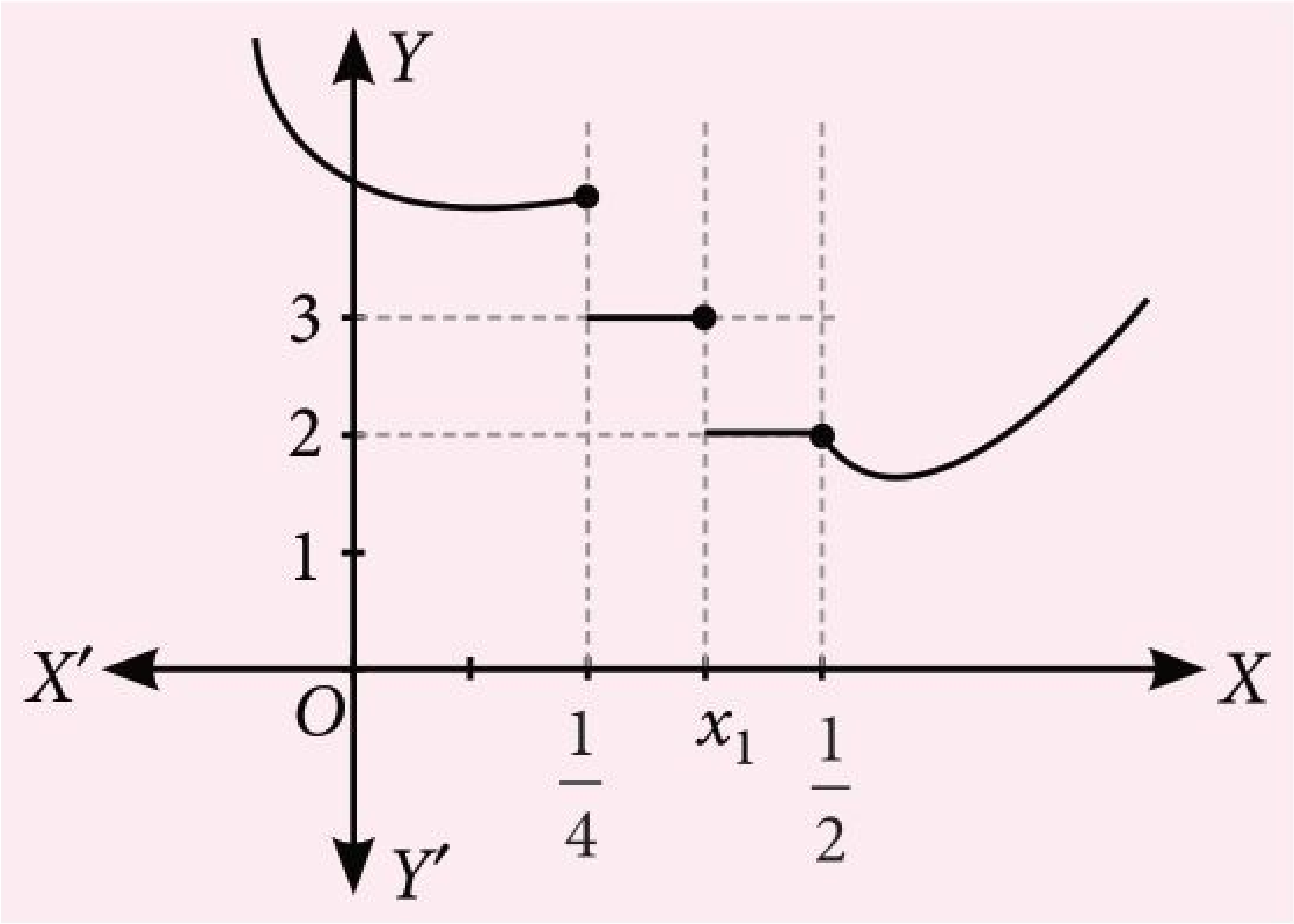
$$=\frac{15^2(16)^2}{2}-\frac{15\times16\times31}{6}=27560$$

26. (3) : We have

$$f(x)=\begin{cases}|4x^2-8x+5|, & \text{if } 8x^2-6x+1\ge 0\\ [4x^2-8x+5], & \text{if } 8x^2-6x+1<0\end{cases}$$

$$f(x)=\begin{cases}|4x^2-8x+5|, & x\in\left(-\infty,\frac{1}{4}\right)\cup\left(\frac{1}{2},\infty\right)\\ [4x^2-8x+5], & x\in\left(\frac{1}{4},\frac{1}{2}\right)\end{cases}$$

$$f(x)=\begin{cases}4x^2-8x+5 & ,\ x\in\left(-\infty,\frac{1}{4}\right)\cup\left(\frac{1}{2},\infty\right)\\ 3 & ,\ x\in\left(\frac{1}{4},x_1\right)\\ 2 & ,\ x\in\left(x_1,\frac{1}{2}\right)\end{cases}$$



Total number of non differentiable points = 3.

27. (5) : We have,

$$\lim_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}\left[(nk+1)+(nk+2)+\ldots+(nk+n)\right]$$

$$=33\cdot\lim_{n\to\infty}\frac{1}{n^{k+1}}\cdot\left[1^k+2^k+3^k+\ldots+n^k\right]$$

Consider L.H.S., $\lim\limits_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}\left[n^2k+\frac{n(n+1)}{2}\right]$

$$=\lim_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}n^2\left[k+\frac{\left(1+\frac{1}{n}\right)}{2}\right]$$

$$=\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{k-1}\left[k+\frac{\left(1+\frac{1}{n}\right)}{2}\right]$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{k-1}\left[k+\frac{\left(1+\frac{1}{n}\right)}{2}\right]=k+\frac{1}{2}$$

Now consider R.H.S.,

$$33\lim_{n\to\infty}\frac{1}{n^{k+1}}\left[1^k+2^k+3^k+\ldots+n^k\right]=33\left(\frac{1}{k+1}\right)$$

Given, L.H.S. = R.H.S.

$$\therefore\quad k+\frac{1}{2}=33\left(\frac{1}{k+1}\right)\Rightarrow k=5,\frac{-13}{2}$$

28. (2) : The equation of two diameters are,

$2px - y = 1$ ...(i)

$2x + py = 4p$ ...(ii)

Solving (i) & (ii), we get

$$x=\frac{5p}{2(p^2+1)},\ y=\frac{4p^2-1}{p^2+1}$$

But from equation of circle, coordinates of centre will be $(1, y)$.

$$\therefore\quad\frac{5p}{2(p^2+1)}=1\ \Rightarrow 2p^2-5p+2=0\ \Rightarrow\ p=2\text{ or }\frac{1}{2}$$

Putting value of p in (i), we get $y = 3$ or $y = 0$

Now, equation of tangent having slope m to the given hyperbola is, $y=mx\pm\sqrt{m^2-3}$

This passes through centre of circle *i.e.*, (1, 3)

$\therefore\quad (3-m)^2=m^2-3$

$\Rightarrow\quad 9+m^2-6m=m^2-3\Rightarrow 6m=12\Rightarrow m=2$

29. (10)

30. (8.33 or 3) : The direction ratios of line of shortest distance is

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 1\\ 2 & 2 & 1\end{vmatrix}=-\hat{i}+2\hat{j}-2\hat{k}.$$

Now, the direction ratios of plane are $<a, -1, -1>$

$$\therefore\quad\cos\theta=\frac{-a-2+2}{\sqrt{1+4+4}\sqrt{a^2+1+1}}$$

$$\Rightarrow\quad\sqrt{\frac{2}{27}}=\frac{-a}{3\sqrt{a^2+2}}\ \Rightarrow\ \frac{2}{27}=\frac{a^2}{9(a^2+2)}\ \Rightarrow a=\pm2$$

For $a = -2$

$P: -2x-y-z=0\ \Rightarrow\ 2x+y+z=0$

The image of point $P(1, 1, -5)$ to the plane P is given as

$$\frac{x-1}{2}=\frac{y-1}{1}=\frac{z+5}{1}=\frac{-2(2+1-5)}{4+1+1}$$

$$\alpha=x=\frac{7}{3},\ \beta=y=\frac{5}{3},\ \gamma=z=\frac{-13}{3}$$

$$\therefore\quad\alpha+\beta-\gamma=\frac{25}{3}$$

For $a = 2$

$P: 2x-y-z=0$

The image of point $P(1, 1, -5)$ to the plane P is given as

$$\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z-1}{-1}=\frac{-2(2-1+5)}{6}$$

$$\alpha=x=-3,\ \beta=y=3,\ \gamma=z=-3\Rightarrow\alpha+\beta-\gamma=3$$

◆◆

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(d)	**4.**	(c)	**5.**	(b)
6.	(a)	**7.**	(b, c)	**8.**	(a, b, c)	**9.**	(a, c)	**10.**	(c, d)
11.	(a, c)	**12.**	(a)	**13.**	(a, b, c)	**14.**	(c)	**15.**	(b)
16.	(d)	**17.**	(139)	**18.**	(1526)	**19.**	(3)	**20.**	(9)

SOLVED PAPER 2022

AMU ENGINEERING

1. Which of the following is the contrapositive of 'if two triangles are identical, then these are similar'?
   (a) If two triangles are not similar, then they are not identical
   (b) If two triangles are not identical, then these are not similar
   (c) If two triangles are not identical, then these are similar
   (d) If two triangles are not similar, then these are identical.

2. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same number on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is
   (a) $\frac{1}{2}$ gain (b) $\frac{1}{4}$ loss
   (c) $\frac{1}{2}$ loss (d) 2 gain

3. The probability when a hand of 7 cards is dealt from a well shuffled deck of 52 cards, it contains all 4 kings is
   (a) $\frac{9}{1547}$ (b) $\frac{1}{1547}$ (c) $\frac{46}{7735}$ (d) $\frac{1}{7735}$

4. Suppose that it is 9 to 7 against a person A who is now 35 years of age living till he is 65, and 3 to 2 against a person *B* now 45 living till he is 75; the chance, that one at least of these two will be alive 30 years hence,
   (a) $\frac{53}{80}$ (b) $\frac{27}{80}$ (c) $\frac{15}{16}$ (d) $\frac{14}{27}$

5. Let n be a two digit natural number. If n is selected at random, then find the probability such that $2^n - 2$ is a multiple of 3.
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 0

6. A bag contains four one-rupee coins, two twenty-five paisa coins and five ten-paisa coins. In how many ways can an amount, not less than Re. 1 be taken out from the bag? (Consider coins of the same denominations to be identical)
   (a) 71 (b) 72 (c) 73 (d) 80

7. If the function
$$f(x)=\begin{cases}\dfrac{\log(1+ax)-\log(1-bx)}{x} & \text{, if } x\neq 0\\ K & \text{, if } x=0\end{cases}$$
   is continuous at $x = 0$, then the value of K is
   (a) $a + b$ (b) $a - b$
   (c) $\frac{a}{b}$ (d) ab

8. If the sum of two unit vectors is a vector of length $\frac{1+\sqrt{3}}{\sqrt{2}}$, then the angle between the two given vectors is
   (a) 0 (b) $\pi/3$ (c) $\pi/2$ (d) $\pi/6$

9. If the function $f(x)=\begin{cases}(\cos x)^{1/x}, & x\neq 0\\ K, & x=0\end{cases}$ is continuous at $x = 0$, then the value of K is
   (a) 0 (b) 1 (c) -1 (d) e

10. If $\vec{a}, \vec{b}$, and $\vec{c}$ are units vectors such that $\vec{a}$ is perpendicular to the plane of $\vec{b}, \vec{c}$ and the angle between $\vec{b}, \vec{c}$ is $\pi/3$, then $|\vec{a}+\vec{b}+\vec{c}|$ is
    (a) 0 (b) 1 (c) 2 (d) 3

11. The general solution of the D.E. $x\frac{dy}{dx}+2y=x^2$, $(x\neq 0)$ is
    (a) $4x^2y = x^4 + c$ (b) $4xy^2 = y^4 + c$
    (c) $4xy = y^4 + c$ (d) $xy = x^2 + cy^3$

12. Let $f : IR \to IR$ be a differentiable function for all values of x and has the property that $f(x)$ and $f'(x)$ have opposite signs for all values of x. Then

(a) $f(x)$ is an increasing function
(b) $f(x)$ is a decreasing function
(c) $f^2(x)$ is a decreasing function
(d) $|f(x)|$ is an increasing function

13. A particular solution of the differential equation $(x - y)(dx + dy) = dx - dy$ given that $y(0) = -1$, is

(a) $\log|x - y| = x + y$
(b) $\log|x - y| = x + y + 1$
(c) $\log|x - y| = x + y - 1$
(d) $\log|x - y| = x - y - 1$

14. Let $f : IR \to IR$ be a differentiable function $\forall\ x \in IR$. If the tangent drawn to the curve at any point $x \in (a, b)$ always lies below the curve, then

(a) $f'(x) > 0, f''(x) < 0\ \forall\ x \in (a, b)$
(b) $f'(x) < 0, f''(x) < 0\ \forall\ x \in (a, b)$
(c) $f'(x) > 0, f''(x) > 0\ \forall\ x \in (a, b)$
(d) None of these

15. A stone is dropped into a quiet lake waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

(a) $10\pi\ \text{cm}^2/\text{s}$ (b) $20\pi\ \text{cm}^2/\text{s}$
(c) $50\pi\ \text{cm}^2/\text{s}$ (d) $100\pi\ \text{cm}^2/\text{s}$

16. If $e^y(x + 1) = 1$, then $\left(\frac{dy}{dx}\right)^2$ is equal to

(a) $\frac{d^2y}{dx^2}$ (b) $\frac{1}{2}\frac{dy}{dx}$
(c) 0 (d) 1

17. The value of $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ is

(a) $\tan(xe^x) + c$ (b) $-\tan(xe^x) + c$
(c) $\operatorname{cosec}(xe^x) + c$ (d) $\sec(xe^x) + c$

18. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then the value of a and b are

(a) $a = 1, b = -1$ (b) $a = -1, b = 1$
(c) $a = 1, b = 1$ (d) $a = -1, b = -1$

19. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-2$
(c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-1$

20. The equation of the plane passing through the intersection of the planes $x + 2y + 3z = 4$ and $4x + 3y + 2z = -1$ and the origin is

(a) $17x - 14y + 11z = 0$
(b) $17x + 14y - 11z = 0$
(c) $-17x + 14y + 11z = 0$
(d) $17x + 14y + 11z = 0$

21. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are

(a) $(1, 0, 2)$ (b) $(2, 0, 1)$
(c) $(-1, 0, 4)$ (d) $(4, 0, -1)$

22. The line $\frac{x}{k} = \frac{y}{2} = \frac{z}{-12}$ makes an isosceles triangle with the planes $2x + y + 3z - 1 = 0$ and $x + 2y - 3z - 1 = 0$ if value of k is

(a) 1 (b) -2 (c) 3 (d) 4

23. The area bounded by the curve $y = \sin^{-1}x$ and the lines $x = 0$, $|y| = \pi/2$ is

(a) 2 (b) 4 (c) 8 (d) 16

24. The area under the curve $y = \frac{(\ln x)}{x^2}$ from $x = 1$ to $x = \infty$ is

(a) 1 (b) ∞ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. The eccentricity of the ellipse $5x^2 + 9y^2 = 1$ is

(a) 1/2 (b) 4/5 (c) 3/4 (d) 2/3

26. A plane meets the co-ordinate axes in A, B and C such that the centroid of the triangle ABC is the point (a, b, c), then equation of the plane is

(a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (b) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$
(c) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$ (d) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=\frac{1}{3}$

27. If $f(x) = 1 + \alpha x, \alpha \neq 0$ is the inverse of itself, then the value of α is

(a) -1 (b) 1 (c) 0 (d) 2

28. The x and y-axes are rotated through an angle $\pi/4$ radians about the origin to get new axes x' and y'. The equation for the hyperbola $2xy = 9$ in the new co-ordinates will be

(a) $x'^2 - y'^2 = 9$ (b) $2x'y' = 9$

(c) $\frac{x'^2}{9} - \frac{y'^2}{9} = 2$ (d) $x'y' = 1$

29. Locus of midpoints of the chords of contact of $x^2 + y^2 = 2$ from the points on the line $3x + 4y = 10$ is a circle with centre P. If O be the origin, then OP is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

30. A point $P(x, y)$ moves in $x - y$ plane such that $x = a\cos^2\theta$ and $y = 2a\sin\theta$, where θ is a parameter. The locus of the point P is

(a) circle (b) ellipse

(c) unbounded parabola

(d) part of the parabola

31. If $P = [x\ y\ z]$, $Q = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $R = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then $[ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx]$ is

(a) $(PQ)R$ (b) $(PR)Q$

(c) $P(RQ)$ (d) none of these

32. The number of 6-digit numbers that can be formed using the three digits 0, 1 and 2 is

(a) 3×2^5 (b) 2×3^5 (c) 3^6 (d) 2×3^6

33. Let α, β, be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If α, β, γ and δ are in G.P., then the integer values of p and q respectively are

(a) $-2, 3$ (b) $-6, 3$

(c) $-6, -32$ (d) $-2, -32$

34. Solving the following equations for x and y

$3x + y - 4z + 5w = 2$

$2x - 3y - 2z + 3w = 5$

with $w > z$ and $0 < w < 1$, we get

(a) $x < 1, y < 0$ (b) $x < 1, y > 0$

(c) $x < 0, y < 0$ (d) $x > 1, y > 0$

35. If ${}^{28}C_{2r} : {}^{24}C_{2r-1} = 225 : 14$, then r is equal to

(a) 24 (b) 14 (c) 7 (d) 21

36. If $A = \begin{pmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{pmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$ $= \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, where I_2 denote the 2×2 identity matrix, then $13(a^2 + b^2)$ is equal to

(a) 1 (b) 0 (c) 13 (d) 2

37. The number of distinct values of θ satisfying $0 \le \theta \le \pi$ and satisfying the equation $\sin\theta + \sin5\theta = \sin3\theta$, is

(a) 6 (b) 7 (c) 8 (d) 9

38. The value of the determinant

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y - a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z - a^{-z})^2 & 1 \end{vmatrix}$$

where $a > 0$ and $x, y, z \in R$ is

(a) $a^x a^y a^z$ (b) 1

(c) 0 (d) $a^x + a^y + a^z$

39. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive then

(a) $abc < -8$ (b) $abc > -8$

(c) $abc > -2$ (d) $abc > 1$

40. Let Z be those complex numbers, which satisfy $|Z + 5| \le 4$ and $Z(1+i) + \bar{Z}(1-i) \ge -10, (i = \sqrt{-1})$.

If the maximum value $|Z + 1|^2$ is $a + b\sqrt{2}$, then the value of $(a + b)$ is

(a) 32 (b) 16 (c) 64 (d) 48

41. If $W = \frac{Z}{Z - \frac{i}{2}}$ and $|W| = 1$, then Z lies on

(a) a parabola (b) a straight line

(c) a circle (d) an ellipse

42. Six boys and six girls sit along a line alternately in x ways and along a circle (again alternately in y ways), then

(a) $x = y$ (b) $y = 12x$

(c) $x = 10y$ (d) $x = 12y$

43. The range of the function $f(x) = [|\sin x| + |\cos x|]$ where $[\cdot]$ denotes the greatest integer function is

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) $\{1, \sqrt{2}\}$

44. If $\sin x + \sin^2 x + \sin^3 x = 1$, then $\cos^6 x - 4\cos^4 x + 8\cos^2 x$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

45. The value of $\int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$ is equal to

(a) -1 (b) 1 (c) 0 (d) e

46. The value of $\sin^{-1}\left(\dfrac{\sin x+\cos x}{\sqrt{2}}\right), \dfrac{-\pi}{4}<x<\dfrac{\pi}{4}$ is

(a) $\pi/4$ (b) $x + \pi/4$
(c) $\pi/4 - x$ (d) x

47. If $f: R \to S$ defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto, then the interval of S is

(a) $[0, 3]$ (b) $[-1, 1]$
(c) $[0, 1]$ (d) $[-1, 3]$

48. Relation R in the set of all integers defined as xRy if and only if $2x + 5y$ is a multiple of 7, then R is

(a) only reflexive
(b) reflexive and symmetric but not transitive
(c) reflexive and transitive but not symmetric
(d) an equivalence relation

49. The following data was observed from a study of car complaints received from 360 respondents at Khan's Car Care Workshop, viz, engine problem, transmission problem or mileage problem. Of those surveyed, there was no one who faced exactly two of these problems. There were 180 respondents who faced engine problems, 240 who faced transmission problems and 300 who faced mileage problems. How many of them faced all the three problems?

(a) 90 (b) 120 (c) 180 (d) 40

50. Let $f: R \to R$ be defined by $f(x)=\dfrac{x}{1+x^2}, x \in R$. Then the range of f is

(a) $(-1, 1) - \{0\}$ (b) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
(c) $R-\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (d) $R - [-1, 1]$

SOLUTIONS

1. (a) : Given, statement is
'If two triangle are identical, then these are similar'.
Let p : Two triangles are identical
q : Two triangles are similar.
In symbolic form $p \to q$
Contrapositive of the given statement is given by $\sim q \to \sim p$.
$\therefore$ If two triangles are not similar, then these are not identical.

2. (c) : Let S = Throwing two dice
$\therefore$ $n(S) = 36$
Let E_1 : Event of getting the same number on the two dice.
E_2 : Event of getting the sum of two numbers on the two dice is 9.
E_3 : Event of any throw other than the above

$\therefore$ $P(E_1)=\dfrac{6}{36}=\dfrac{1}{6}, P(E_2)=\dfrac{4}{36}=\dfrac{1}{9}, P(E_3)=\dfrac{26}{36}=\dfrac{13}{18}$

$\therefore$ Expected loss/gain $=15\times\dfrac{1}{6}+12\times\dfrac{1}{9}-6\times\dfrac{13}{18}$

$=\dfrac{1}{2}$ (loss)

3. (d) : Total number of possible hand = $^{52}C_7$.
Number of cards having all 4 kings = $^4C_4 \times {}^{48}C_3$

$\therefore$ Required probability = $\dfrac{^4C_4 \times {}^{48}C_3}{^{52}C_7}=\dfrac{1}{7735}$

4. (a) : We have, $P(A)=\dfrac{7}{16} \Rightarrow P(\bar{A})=\dfrac{9}{16}$

Also, $P(B)=\dfrac{2}{5} \Rightarrow P(\bar{B})=\dfrac{3}{5}$

$\therefore$ Required probability

$=P(A)\cdot P(\bar{B})+P(\bar{A})\cdot P(B)+P(A)\cdot P(B)$

$=\dfrac{7}{16}\times\dfrac{3}{5}+\dfrac{9}{16}\times\dfrac{2}{5}+\dfrac{7}{16}\times\dfrac{2}{5}=\dfrac{21+18+14}{80}=\dfrac{53}{80}$

5. (b) : Total two digit numbers are 90.
Now, $2^n - 2 = (3 - 1)^n - 2$
$= {}^nC_0\, 3^n \cdot (-1)^0 + {}^nC_1\, 3^{n-1} \cdot (-1)^1 + {}^nC_2\, 3^{n-2}(-1)^2 + \ldots.. + {}^nC_0\, 3^0 (-1)^n - 2$
$= (3^n - {}^nC_1\, 3^{n-1} + {}^nC_2\, 3^{n-2} + \ldots..) + (-1)^n - 2$
$= 3(K) + (-1)^n - 2$
$\therefore$ $2^n - 2$ is divisible when n is odd.
Total two digits odd numbers = 45.

$\therefore$ Required probability = $\dfrac{45}{90}=\dfrac{1}{2}$

6. (c) : Case-I : If we do not select 1 rupee coin. Then to make 1 rupee, we have only one way to select other coins that is two twenty-five paisa coins and five ten-paisa coins.
Case-II : If we select atleast one coin of 1 rupee, then total ways = (3 + 1) (2 + 1) (5 + 1) ways = 72 ways
$\therefore$ Required number of ways = (72 + 1) ways = 73 ways.

7. (a) : We have,

$$f(x)=\begin{cases}\dfrac{\log(1+ax)-\log(1-bx)}{x}, & \text{if } x \neq 0\\ K, & \text{if } x=0\end{cases}$$

Since $f(x)$ is continuous at $x = 0$.

$$\therefore \lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0} \frac{\log(1+ax)-\log(1-bx)}{x} = K$$

$$\Rightarrow \lim_{x\to 0}\left\{\frac{\log(1+ax)}{ax}\times a - \frac{\log(1-bx)}{-bx}\times -b\right\} = K$$

$\therefore\ K = a + b$

8. (d) : Let $\vec{a}$ and $\vec{b}$ be two unit vectors.

$|\vec{a}| = |\vec{b}| = 1$

Let the third vector be $\vec{c}$, whose length is $\dfrac{1+\sqrt{3}}{\sqrt{2}}$.

$\therefore\ \vec{c} = \vec{a}+\vec{b}\ ;\ |\vec{c}|^2 = |\vec{a}+\vec{b}|^2$

$$\Rightarrow \left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$\Rightarrow \left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)^2 = 1+1+2\times1\times1\cos\theta.$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}\ ;\ \theta = \frac{\pi}{6}$$

9. (b) : We have, $f(x) = \begin{cases} (\cos x)^{1/x}, & x \neq 0 \\ K, & x = 0 \end{cases}$

$f(x)$ is continuous at $x = 0$.

$$\lim_{x\to 0} f(x) = f(0)\ ;\ \lim_{x\to 0}(\cos x)^{1/x} = K$$

$$\Rightarrow K = e^{\left(\frac{\cos x - 1}{x}\right)} = e^{\frac{-2\sin^2 x/2}{x}} \Rightarrow e^0 = K$$

$\therefore\ K = 1$

10. (c) : We have, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Given, $\vec{a}\cdot\vec{b} = 0, \vec{a}\cdot\vec{c} = 0$

Now,

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})}$$

Now, $= \sqrt{1+1+1+2\times1\times1\times\frac{1}{2}} = 2$

11. (a) : Given differential equation is $x\dfrac{dy}{dx} + 2y = x^2$

$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

Solution is given as

$$y\cdot(x)^2 = \int x^2\cdot x\,dx + C \Rightarrow x^2 y = \frac{x^4}{4} + C$$

$\Rightarrow 4x^2y = x^4 + c.$

12. (c) : Given that $f(x)$ and $f'(x)$ have opposite sign.

$\therefore\ f(x)\cdot f'(x) < 0.$

We can not say that $f(x)$ is increasing or decreasing.

Consider option (c).

$$\frac{d}{dx}\{f^2(x)\} = 2f(x)\cdot f'(x) < 0$$

$\therefore$ Option (c) is correct.

13. (b) : Given D.E is $(x - y)(dx + dy) = dx - dy$.

$$\Rightarrow dx + dy = \frac{dx - dy}{x - y}$$

Integrating both sides. $x + y = \log|x - y| + c$

Put $x = 0, y = -1$

$0 + (-1) = \log|0 + 1| + c \Rightarrow c = -1$

$\therefore$ Solution is $x + y = \log|x - y| - 1$

14. (d)

15. (d) : Let r be the radius of the circular wave.

$$\therefore \frac{dr}{dt} = 5\text{ cm/s}$$

Area, $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r\cdot\frac{dr}{dt} = 2\pi\cdot 10\times 5 = 100\,\pi\text{ cm}^2/\text{sec}$$

16. (a) : We have, $e^y(x + 1) = 1$.

$$e^y = \frac{1}{x+1} \Rightarrow y = -\ln(x+1) \qquad \ldots\text{(i)}$$

Differentiating (i) w.r.t 'x', we get

$$y' = \frac{-1}{x+1}\ ;\ (y')^2 = \frac{1}{(x+1)^2} = y''$$

$$\therefore \left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$$

17. (a) : Let $I = \displaystyle\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$

Let $xe^x = t \Rightarrow e^x(1 + x)dx = dt$

$$\therefore I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\,dt = \tan t + c = \tan(xe^x) + c$$

18. (b) : We have $\displaystyle\int_a^b x^3dx = 0, \int_a^b x^2dx = \frac{2}{3}$

$f_1 = x^3, f_2 = x^2.$

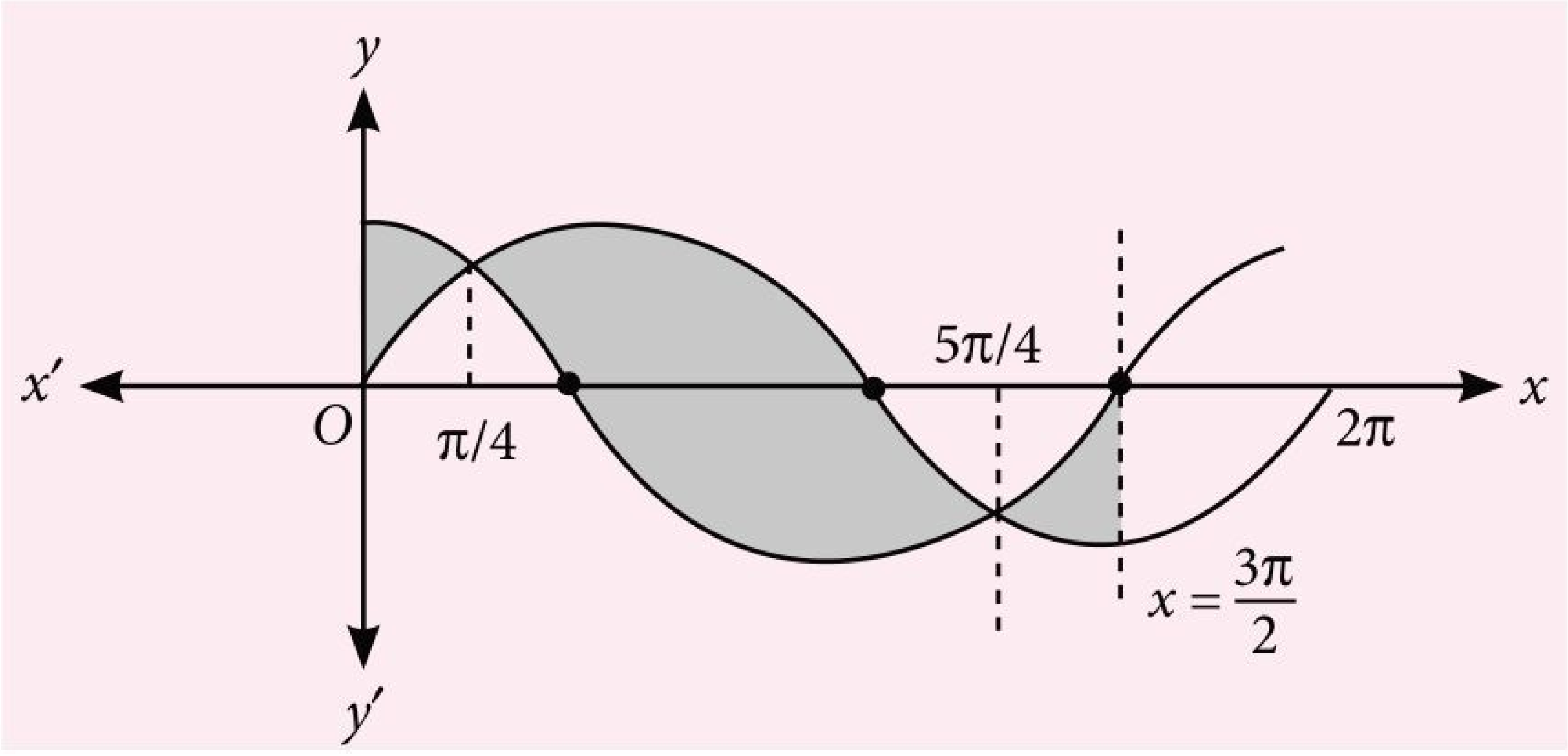
f_1 is odd and f_2 is even function.

$\therefore$ If $a = -1$ and $b = 1$, then

$$\int_{-1}^{1} x^3dx = 0 \text{ and } \int_{-1}^{1} x^2dx = 2\int_0^1 x^2dx = \frac{2}{3}$$

19. (b) : Given curves are $y = \cos x$, $y = \sin x$

$x = 0$, and $x = \dfrac{3\pi}{2}$.



$$\therefore \text{ Required area} = \int_0^{\pi/4} (\cos x - \sin x)\,dx$$

$$+ \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)\,dx$$

$$= 2\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$$

$$= 2\left[\sin x + \cos x\right]_0^{\pi/4} + (-1)\left[\cos x + \sin x\right]_{\pi/4}^{5\pi/4}$$

$$= 2\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] - \left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= (4\sqrt{2} - 2) \text{ sq. units}$$

20. (d) : Equation of plane passing through the intersection of planes $x + 2y + 3z = 4$ and $4x + 3y + 2z = -1$ is given by

$x + 2y + 3z - 4 + \lambda(4x + 3y + 2z + 1) = 0$

$(1 + 4\lambda)x + (2 + 3\lambda)y + (3 + 2\lambda)z + (\lambda - 4) = 0$ ...(i)

Since, it passes through (0, 0, 0).

$\therefore\ \lambda - 4 = 0 \Rightarrow \lambda = 4$

Put $\lambda = 4$ in (i), we get $17x + 14y + 11z = 0$,

Which is equation of required plane.

21. (b) : Given, $L : \dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z}{1}$

$P_1 : x + y + z = 3.$

$P_2 : x - y + z = 3.$

Let any point P on the line L is $(2\lambda + 1, -\lambda - 1, \lambda)$.

Let $Q \equiv (\alpha, \beta, \gamma)$

Dr's of line PQ are $< \alpha - 2\lambda - 1, \beta + \lambda + 1, \gamma - \lambda >$

Dr's of plane P_1 are <1, 1, 1>

$\alpha - 2\lambda - 1 = 1, \beta + \lambda + 1 = 1 ; \gamma - \lambda = 1$

$\Rightarrow\ \alpha = 2\lambda + 2, \beta = -\lambda ; \gamma = \lambda + 1$

Put these values in plane P_2,

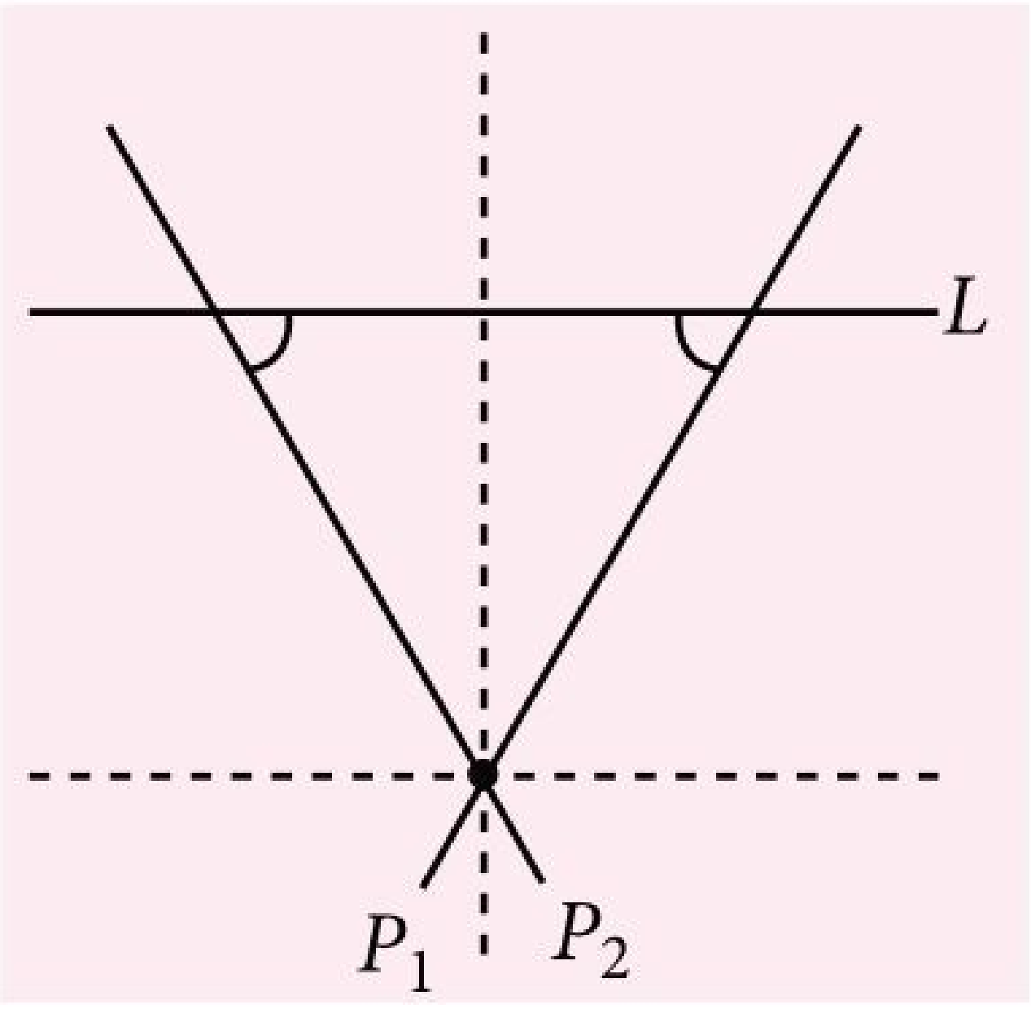
we get $\lambda = 0$

$\therefore\ Q \equiv (2, 0, 1).$

22. (b) : Given, $L : \dfrac{x}{k} = \dfrac{y}{2} = \dfrac{z}{-12}$

$P_1 : 2x + y + 3z - 1 = 0$

$P_2 : x + 2y - 3z - 1 = 0.$



As the line L makes isosceles triangle with P_1 and P_2. Then the angle bisector of P_1 and P_2 will be parallel to the line L.

So, angle bisector of P_1 and P_2 is $\dfrac{2x + y + 3z - 1}{\sqrt{4+1+9}} = \pm \dfrac{x + 2y - 3z - 1}{\sqrt{1+4+9}}$

$\Rightarrow 2x + y + 3z - 1 = \pm(x + 2y - 3z - 1)$

$\Rightarrow 2x + y + 3z - 1 = x + 2y - 3z - 1$

or $2x + y + 3z - 1 = -x - 2y + 3z + 1$

$\Rightarrow x - y + 6z = 0$ ...(i)

or $3x + 3y - 2 = 0$ ...(ii)

Line (i) is parallel to L

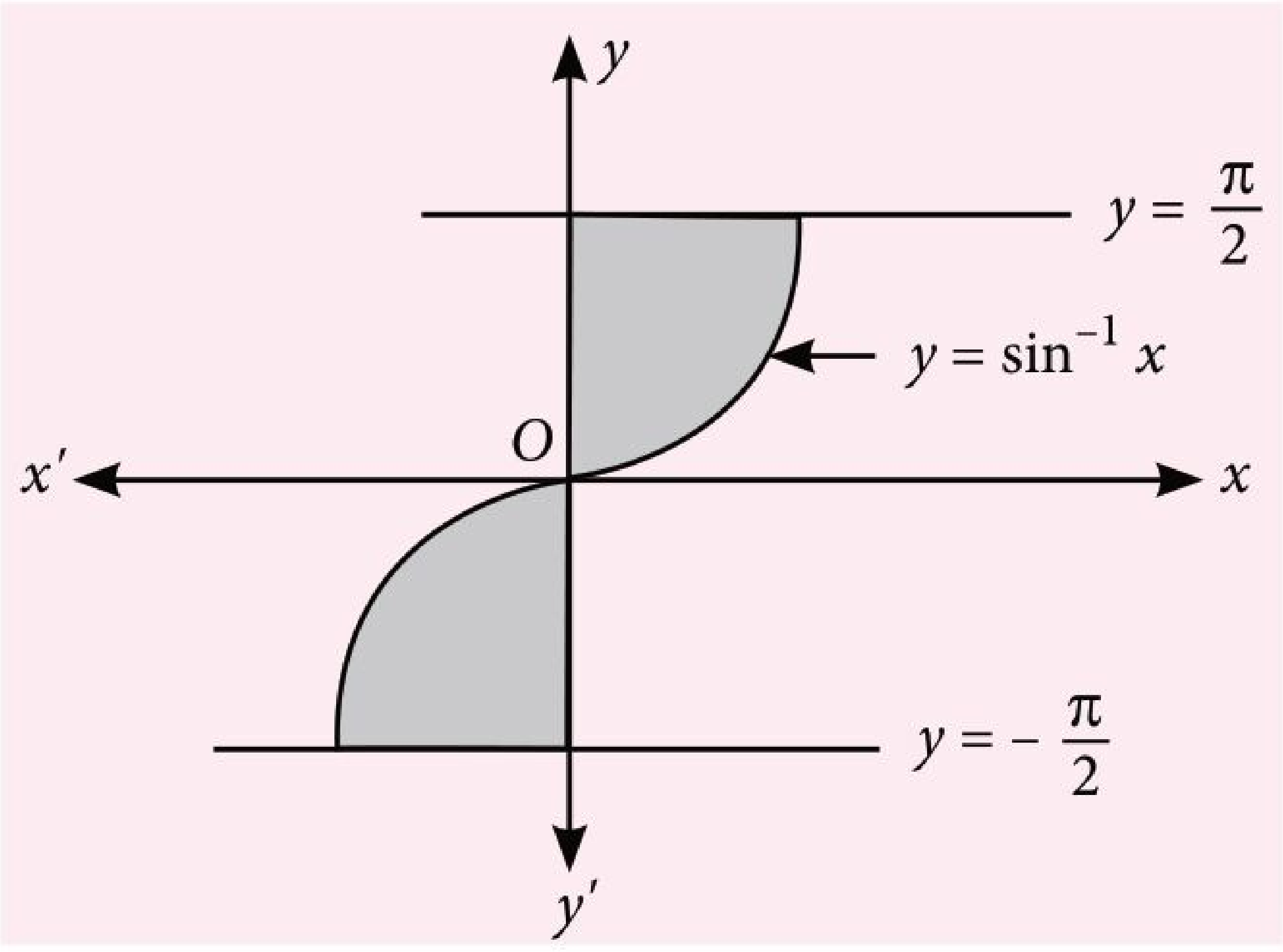
$\dfrac{k}{1} = \dfrac{2}{-1} = \dfrac{-12}{6}$; $\dfrac{k}{1} = -2$

SAMURAI SUDOKU

ANSWER - AUGUST 2022

6	7	5	2	1	4	8	9	3				3	7	5	2	8	1	4	6	9
2	3	1	5	8	9	6	7	4				1	4	8	9	7	6	5	2	3
9	8	4	7	3	6	2	1	5				9	2	6	4	3	5	7	1	8
4	9	6	8	7	5	3	2	1				5	6	3	1	2	8	9	7	4
3	2	8	9	6	1	4	5	7				4	1	7	6	9	3	8	5	2
5	1	7	3	4	2	9	6	8				2	8	9	5	4	7	6	3	1
1	4	2	6	5	3	7	8	9	2	1	3	6	5	4	8	1	2	3	9	7
8	5	9	4	2	7	1	3	6	8	5	4	7	9	2	3	6	4	1	8	5
7	6	3	1	9	8	5	4	2	7	9	6	8	3	1	7	5	9	2	4	6
						6	5	8	3	2	9	4	1	7						
						9	1	7	6	4	8	3	2	5						
						4	2	3	5	7	1	9	6	8						
4	9	2	6	8	5	3	7	1	9	8	5	2	4	6	5	3	9	1	7	8
7	5	6	2	3	1	8	9	4	1	6	2	5	7	3	8	1	2	9	6	4
3	8	1	9	4	7	2	6	5	4	3	7	1	8	9	7	4	6	3	2	5
2	4	8	5	7	9	6	1	3				7	6	4	2	9	1	8	5	3
1	7	3	8	6	2	4	5	9				9	1	5	3	6	8	7	4	2
9	6	5	4	1	3	7	8	2				8	3	2	4	5	7	6	1	9
6	1	7	3	5	4	9	2	8				6	5	1	9	2	3	4	8	7
5	2	4	7	9	8	1	3	6				3	2	7	1	8	4	5	9	6
8	3	9	1	2	6	5	4	7				4	9	8	6	7	5	2	3	1

23. (a) : Given curves are, $y = \sin^{-1} x$, $x = 0$, $y = \pm\frac{\pi}{2}$



$$\text{Required area} = 2\int_0^{\pi/2} x\,dy$$

$$= 2\int_0^{\pi/2} \sin y \cdot dy \;\; = -2\left[\cos y\right]_0^{\pi/2} \Rightarrow -2[0-1] = 2$$

24. (a) : Area $= \int_1^{\infty} \frac{\ln x}{x^2} dx$

$$= \left[(\ln x)\cdot\left(\frac{-1}{x}\right)\right]_1^{\infty} + \int_1^{\infty} \frac{1}{x}\cdot\frac{1}{x} dx \;= (0-0) + \left(\frac{-1}{x}\right)_1^{\infty}$$

$= 0 - (-1) = 1$

25. (d) : Given ellipse can be written as

$$\frac{x^2}{1/5} + \frac{y^2}{1/9} = 1 \;;\; \text{Here, } a^2 = \frac{1}{5}, b^2 = \frac{1}{9}$$

Now, $e^2 = 1 - \frac{5}{9} = \frac{4}{9} \;\Rightarrow\; e = \frac{2}{3}$

26. (c) : Let $A \equiv (\alpha, 0, 0)$, $B \equiv (0, \beta, 0)$ $C \equiv (0, 0, \gamma)$.

Centroid of triangle is (a, b, c).

$$a = \frac{\alpha+0+0}{3}, b = \frac{0+\beta+0}{3}, c = \frac{0+0+\gamma}{3}$$

$$\therefore\; a = \frac{\alpha}{3}, b = \frac{\beta}{3}, c = \frac{\gamma}{3}$$

Equation of plane passing through A, B and C is

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1 \Rightarrow \frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1 \;\Rightarrow\; \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$

27. (a) : We have, $f(x) = 1 + \alpha x = y$

$$\Rightarrow\; x = \frac{y-1}{\alpha} \quad \therefore\; f^{-1}(y) = \frac{y-1}{\alpha} \;\Rightarrow\; f^{-1}(x) = \frac{x-1}{\alpha}$$

Since, $f(x) = f^{-1}(x)$; $1 + \alpha x = \frac{x-1}{\alpha}$

$$\Rightarrow \alpha x + 1 = \left(\frac{1}{\alpha}\right)x - \frac{1}{\alpha} \;;\; \alpha = \frac{1}{\alpha} \text{ and } \frac{-1}{\alpha} = 1$$

$\Rightarrow \alpha^2 = 1$ and $\alpha = -1$

$\therefore\; \alpha = -1$

28. (a) : Given hyperbola is $2xy = 9$.

$$x' = x\cos\frac{\pi}{4} + y\sin\frac{\pi}{4},\; y' = y\cos\frac{\pi}{4} - x\sin\frac{\pi}{4}$$

$$x' = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}},\; y' = \frac{y}{\sqrt{2}} - \frac{x}{\sqrt{2}}$$

$\therefore\; 2xy = 9$

$$\Rightarrow\; 2\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) = 9 \;\Rightarrow\; (x')^2 - (y')^2 = 9.$$

29. (a) : Circle : $x^2 + y^2 = 2$.

Line : $3x + 4y = 10$

Let the mid point of chord of contact is $M(\alpha, \beta)$.

$\therefore\; \alpha x + \beta y - 2 = \alpha^2 + \beta^2 - 2$

Equation of the chord of contact is given by

$x\alpha + y\beta - 2 = \alpha^2 + \beta^2 - 2$

$\Rightarrow x\alpha + y\beta = \alpha^2 + \beta^2$. ...(i)

Let the point on the line $3x + 4y = 10$ is (x_1, y_1).

Then the equation of chord of contact from point (x_1, y_1) is given by

$xx_1 + yy_1 = 2$ ...(ii)

Comparing equations (i) and (ii), we get

$$\frac{\alpha}{x_1} = \frac{\beta}{y_1} = \frac{\alpha^2+\beta^2}{2} \;;\; x_1 = \frac{2\alpha}{\alpha^2+\beta^2}, y_1 = \frac{2\beta}{\alpha^2+\beta^2}$$

Since the point (x_1, y_1) lies on the line $3x + 4y = 10$.

$$\therefore\; 3\left(\frac{2\alpha}{\alpha^2+\beta^2}\right) + 4\left(\frac{2\beta}{\alpha^2+\beta^2}\right) = 10$$

$$6\alpha + 8\beta = 10(\alpha^2 + \beta^2) \;;\; \alpha^2 + \beta^2 - \frac{3}{5}\alpha - \frac{4}{5}\beta = 0$$

$\therefore$ The locus of the point (α, β) is

$$x^2 + y^2 - \frac{3}{5}x - \frac{4}{5}y = 0,$$

which is circle with centre $\left(\frac{3}{10}, \frac{2}{5}\right)$.

$$\therefore\; OP = \sqrt{\frac{9}{100} + \frac{4}{25}} = \frac{1}{2}$$

30. (c) : We have, $x = a\cos^2\theta$, $y = 2a\sin\theta$

$$\Rightarrow \cos^2\theta = \frac{x}{a}, \sin^2\theta = \frac{y^2}{4a^2}$$

Now, $\cos^2\theta + \sin^2\theta = \dfrac{x}{a} + \dfrac{y^2}{4a^2}$

∴ $y^2 = -4a(x - a)$, which is the equation of parabola.

31. (a)

32. (b) :

6	5	4	3	2	1

6th place can be filled only by 1 or 3.
But the remaining places can be filled by 0 or 1 or 2.
Here repetition is allowed.
∴ Required number of ways = 2×3^5.

33. (d) : α, β, γ and δ are in G.P.
$\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$
$\alpha + \beta = 1 \Rightarrow a + ar = 1 \Rightarrow a(1 + r) = 1$ ...(i)
Also, $\gamma + \delta = 4 \Rightarrow ar^2 + ar^3 = 4 \Rightarrow ar^2(1 + r) = 4$ ...(ii)

From (i) and (ii), $\dfrac{1}{r^2} = \dfrac{1}{4} \Rightarrow r = \pm 2$

When $r = 2 \Rightarrow a = \alpha = \dfrac{1}{3}$

$$\beta = ar = \frac{1}{3}\times 2 = \frac{2}{3} \quad \therefore\ \alpha\beta = p$$

$$\frac{1}{3}\times\frac{2}{3} = p\ ;\ p = \frac{2}{9}\ ;\ \gamma = ar^2 = \frac{1}{3}\times 4 = \frac{4}{3}$$

When $r = -2$
$a = \alpha = -1; \beta = ar = -1 \times -2 = 2\ \therefore p = -2$
$\gamma = ar^2 = (-2)^2 \times (-1) = -4, \delta = ar^3 = (-1)(-2)^3 = 8$
∴ $\gamma\delta = -32 = q$.

34. (a)

35. (c) : We have, $\dfrac{^{28}C_{2r}}{^{24}C_{2r-1}} = \dfrac{225}{14}$

$$\Rightarrow \frac{\underline{|28}}{\underline{|28-2r}\,\underline{|2r}} \times \frac{\underline{|2r-1}\times\underline{|25-2r}}{\underline{|24}} = \frac{225}{14}$$

$$\Rightarrow \frac{28\times27\times26\times25}{(28-2r)(27-2r)(26-2r)2r} = \frac{225}{14}$$

$\Rightarrow r = 7$ satisfies the equation.

36. (c)

37. (a) : We have, $\sin\theta + \sin 5\theta = \sin 3\theta$
$\Rightarrow 2\sin 3\theta \cdot \cos 2\theta = \sin 3\theta \Rightarrow \sin 3\theta\,(2\cos 2\theta - 1) = 0$

$$\Rightarrow \sin 3\theta = 0 \text{ or } \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{3} \text{ or } \theta = n\pi \pm \frac{\pi}{6}$$

So, $\theta = 0, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \pi, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ $[\because\ \theta \in (0, \pi)]$

38. (c) : Let $\Delta = \begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y - a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z - a^{-z})^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, we get

$$\Delta = \begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (a^y - a^{-y})^2 & 1 \\ 4 & (a^z - a^{-z})^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - 4C_3$, we get

$$\Delta = \begin{vmatrix} 0 & (a^x - a^{-x})^2 & 1 \\ 0 & (a^y - a^{-y})^2 & 1 \\ 0 & (a^z - a^{-z})^2 & 1 \end{vmatrix} \Rightarrow \Delta = 0$$

39. (b) : Let $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$

$\because\ \Delta$ is positive.
$\Rightarrow \Delta > 0 \Rightarrow a(bc - 1) - 1(c - 1) + 1(1 - b) > 0$
$\Rightarrow abc - a - c + 1 + 1 - b > 0$
$\Rightarrow abc + 2 > a + b + c$ ...(i)
Now, $\dfrac{a+b+c}{3} \ge \sqrt[3]{abc} \Rightarrow (a + b + c)^3 \ge 27abc$
From (i), we have
$(abc + 2)^3 > 27abc$
$\Rightarrow (abc)^3 + 8 + 6(abc)^2 - 15abc > 0$
$\Rightarrow (abc - 1)\,[(abc)^2 + 7(abc) - 8) > 0$
$\Rightarrow (abc - 1)^2\,(abc + 8) > 0 \Rightarrow abc > -8$

40. (d)

41. (b) : We have, $W = \dfrac{Z}{Z - i/2}$
Let $Z = x + iy$
Also, $|W| = 1$

$$\Rightarrow \frac{|Z|}{\left|Z - \frac{i}{2}\right|} = 1 \Rightarrow |Z|^2 = \left|Z - \frac{i}{2}\right|^2$$

$$\Rightarrow\ x^2+y^2=x^2+\left(y-\frac{1}{2}\right)^2\ \Rightarrow\ y=\frac{1}{4},$$

which is a straight line.

42. (d) : No. of boys and girls are 6 each.

$\therefore$ Number of ways to arrange them in a line = 2 × (6! × 6!)

$\Rightarrow x = 2 \times (6!)^2$

Also, number of ways to arrange them in a circle= 6! × 5!

$\Rightarrow y = 6! \times 5!$

So, $x = 2\cdot 6\cdot 6!\cdot 5!$

$\Rightarrow x = 12y$

43. (b) : We have $1\le|\sin x|+|\cos x|\le\sqrt{2}$

$\therefore$ Minimum value of $[|\sin x| + |\cos x|] = [1] = 1$

and maximum value of $[|\sin x| + |\cos x|] = [\sqrt{2}]=1$

$\therefore$ Range of $f(x) = \{1\}$

44. (d) : We have, $\sin x + \sin^2x + \sin^3x = 1$

$\Rightarrow \sin x\,(1 + \sin^2x) = 1 - \sin^2x$

Squaring both sides, we get

$\sin^2x\,(1 + \sin^2x)^2 = (\cos^2x)^2$

$\Rightarrow (1 - \cos^2x)\,(1 + 1 - \cos^2x)^2 = \cos^4x$

$\Rightarrow (1 - \cos^2x)\,(4 + \cos^4x - 4\cos^2x) = \cos^4x$

$\Rightarrow \cos^6x - 4\cos^4x + 8\cos^2x = 4$

45. (b) : We have, $I=\int_{1/e}^{\tan x}\frac{t\,dt}{1+t^2}+\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$

$$=\frac{1}{2}\int_{1/e}^{\tan x}\frac{2t\,dt}{1+t^2}+\int_{1/e}^{\cot x}\left(\frac{1}{t}-\frac{t}{1+t^2}\right)dt$$

$$=\frac{1}{2}\Big[\log|1+t^2|\Big]_{1/e}^{\tan x}+\Big[\log|t|\Big]_{1/e}^{\cot x}-\frac{1}{2}\Big[\log|1+t^2|\Big]_{1/e}^{\cot x}$$

$$=\frac{1}{2}\log(\sec^2 x)+\log(\cot x)-\log\left(\frac{1}{e}\right)-\frac{1}{2}\log(\operatorname{cosec}^2 x)$$

$= \log(\sec x) + \log(\cos x) - \log\sin x + 1 - \log(\operatorname{cosec} x)$

$= 1$

46. (b) : We have, $\sin^{-1}\left(\dfrac{\sin x+\cos x}{\sqrt{2}}\right)$

$$=\sin^{-1}\left(\cos\frac{\pi}{4}\sin x+\cos x\cdot\sin\frac{\pi}{4}\right)$$

$$=\sin^{-1}\left(\sin\left(x+\frac{\pi}{4}\right)\right)=x+\frac{\pi}{4}$$

47. (d) : We have, $-\sqrt{1+3}\le\sin x-\sqrt{3}\cos x\le\sqrt{1+3}$

$\Rightarrow\ -2\le\sin x-\sqrt{3}\cos x\le 2$

$\Rightarrow\ -1\le\sin x-\sqrt{3}\cos x+1\le 3\qquad\therefore\ S = [-1, 3]$

48. (d) : Reflexive : $\because\ 2x + 5x = 7x$ is a multiple of 7.

$\therefore$ R is reflexive

Symmetric : let $xRy \Rightarrow 2x + 5y = 7k$ (say)

$\Rightarrow (7x + 7y) - (2x + 5y) = 7\lambda\qquad$ [where $\lambda = k + x + y$]

$\Rightarrow 5x + 2y = 7\lambda \Rightarrow yRx$

$\therefore$ R is symmetric

Transitive : Let xRy and yRz

$\Rightarrow 2x + 5y = 7k_1$ and $2y + 5z = 7k_2$

Adding above two equations, we get

$2x + 5y + 2y + 5z = 7\,(k_1 + k_2)$

$\Rightarrow 2x + 7y + 5z = 7(k_1 + k_2)$

$\Rightarrow 2x + 5z = 7\lambda$, where $\lambda = k_1 + k_2 - y \Rightarrow xRz$

$\therefore$ R is transitive

So, R is an equivalence relation.

49. (c)

50. (b) : Let $y=\dfrac{x}{1+x^2}$

$\Rightarrow y+yx^2=x\ \Rightarrow x^2y-x+y=0$

$\because\ x\in R\qquad\therefore\ D\ge 0$

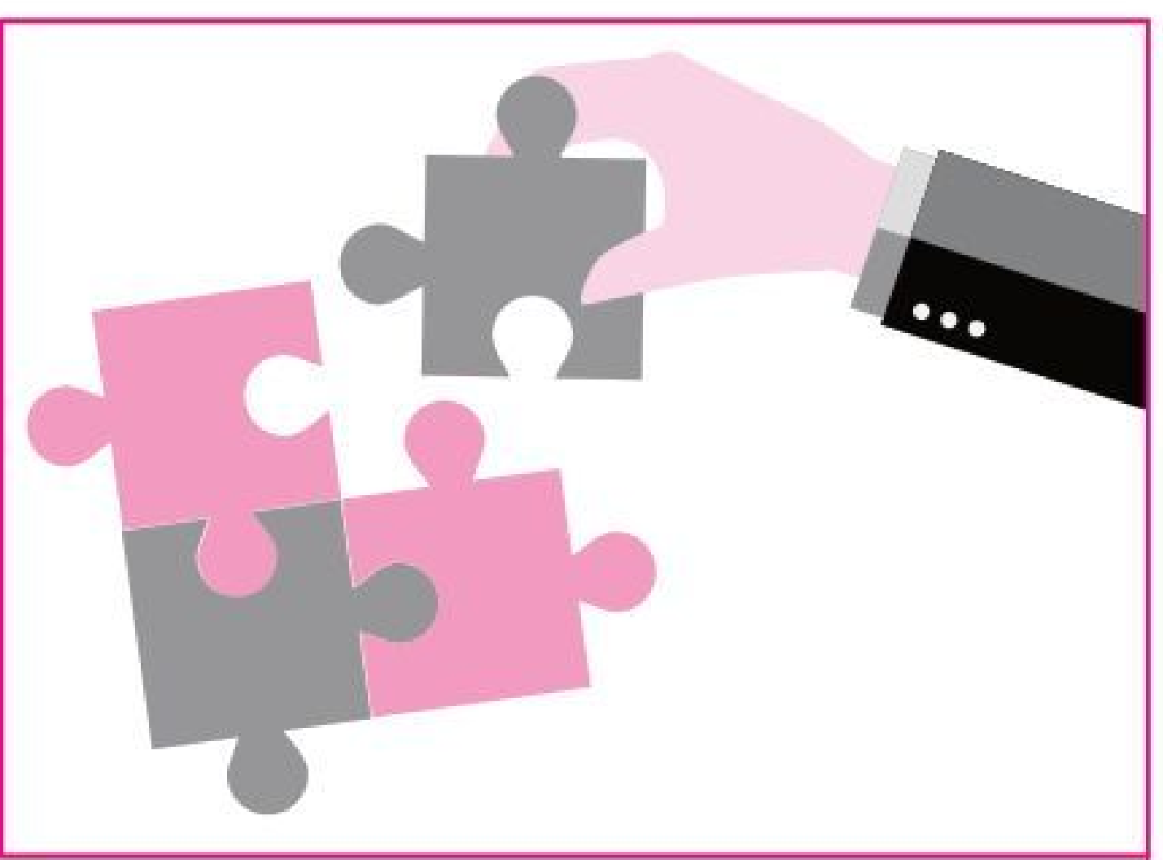
$\Rightarrow 1-4y^2\ge 0\ \Rightarrow\ y^2\le\frac{1}{4}\ \Rightarrow\ \frac{-1}{2}\le y\le\frac{1}{2}$

So, range of $f=\left[\frac{-1}{2},\frac{1}{2}\right]$

* * *

PUZZLE CORNER

ANSWER - AUGUST 2022



450× 5	6	2÷ 4	2	6+ 1	3
3	5	3÷ 6	1– 4	2	3– 1
10+ 6	1 1	2	3	12+ 5	4
1	3	16+ 5	6	4	13+ 2
2÷ 4	3+ 2	1	5	3	6
2	1– 4	3	5– 1	6	5

CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

SERIES - I Relations and Functions | Inverse Trigonometric Functions

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time Allowed : 45 Minutes

Maximum Marks : 200

Multiple Choice Questions (MCQs)

1. Let $X = \{-1, 0, 1\}$, $Y = \{0, 2\}$ and a function $f : X \to Y$ defined by $y = 2x^4$, is

(a) one-one onto
(b) one-one into
(c) many-one onto
(d) many-one into

2. If $y = \sec^{-1}x$, then

(a) $0 \le y \le \pi$
(b) $0 \le y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y \le \pi$
(c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$
(d) None of these

3. The number of commutative binary operations that can be defined on a set of 2 elements is

(a) 8
(b) 6
(c) 4
(d) 2

4. The domain of the function $\cos^{-1}(2x - 1)$ is

(a) $[0, 1]$
(b) $[-1, 1]$
(c) $(-1, 1)$
(d) $[0, \pi]$

5. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ is such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $[ho(gof)]\,(x)$, if $x = \frac{\sqrt{\pi}}{2}$ will be

(a) 0
(b) 1
(c) -1
(d) 10

6. The domain of the function defined by $f(x) = \sin^{-1}x + \cos x$ is

(a) $[-1, 1]$
(b) $[-1, \pi + 1]$
(c) $(-\infty, \infty)$
(d) ϕ

7. Which one of the following relations on R is an equivalence relation?

(a) $aR_1b \Leftrightarrow |a| = |b|$
(b) $aR_2b \Leftrightarrow a \ge b$
(c) $aR_3b \Leftrightarrow a$ divides b
(d) $aR_4b \Leftrightarrow a < b$

8. The value of $\sin^{-1}[\cos(\sin^{-1}x)] + \cos^{-1}[\sin(\cos^{-1}x)]$ is

(a) 0
(b) $\pi/4$
(c) $\pi/2$
(d) π

9. The range of the function

$$f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)....\infty}}} \text{ is}$$

(a) $[0, 1]$
(b) $(0, 1)$
(c) $[0, 2]$
(d) $(0, 2)$

10. $\cot^{-1}(2\cdot1^2) + \cot^{-1}(2\cdot2^2) + \cot^{-1}(2\cdot3^2) + ...$ upto $\infty =$

(a) $\pi/2$
(b) $\pi/5$
(c) $\pi/4$
(d) $\pi/3$

11. Let $f : [0, \infty) \to [0, 2]$ be defined by $f(x) = \frac{2x}{1+x}$, then f is

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

12. If $\sin^{-1}x \cos^{-1}x = p$, then the value of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ is

(a) $\left(\frac{\pi}{2}\right)\left(\frac{\pi}{4} - 3p\right)$
(b) $\left(\frac{\pi}{4}\right)\left(\frac{\pi^2}{4} - 3p\right)$
(c) $\left(\frac{\pi}{2}\right)\left(\frac{\pi^2}{4} - 3p\right)$
(d) $\left(\frac{\pi}{4}\right)\left(\frac{\pi^2}{2} - 3p\right)$

13. Let $*$ be a binary operation on set of integers I defined by $a * b = 2a + b - 3$, then find the value of $3 * 4$.

(a) 2
(b) 4
(c) 7
(d) 6

14. A relation R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relative prime to y. Then, domain of R is

(a) $\{2, 3, 5\}$
(b) $\{3, 5\}$
(c) $\{2, 3, 4\}$
(d) $\{2, 3, 4, 5\}$

15. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then $x^4 + y^4 + z^4 + 4x^2y^2z^2 =$

(a) $x^2 + y^2 + y^2z^2$
(b) $2(x^2y^2 + y^2z^2 + z^2x^2)$
(c) $(x+y)^2$
(d) $(x + y + z)^2$

16. If $A = \{a, b, c\}$ and $B = \{-2, -1, 0, 1, 2\}$, then total number of one-one functions from A to B is
(a) 60 (b) 30 (c) 15 (d) 20

17. If $4\cos^{-1}x + \sin^{-1}x = \pi$, then the value of x is
(a) $\frac{3}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

18. Let $f(x) = 2^{10}x + 1$ and $g(x) = 3^{10}x - 1$. If $fog(x) = x$, then x is equal to
(a) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\frac{2^{10}+1}{2^{10}-3^{-10}}$ 000
(c) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$ (d) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

19. The value of $2\sin^{-1}\sqrt{\frac{1-x}{2}}$ is
(a) $\cos^{-1}x$ (b) $2\cos^{-1}\sqrt{\frac{1+x}{2}}$
(c) Both (a) and (b) (d) None of these

20. Let $f(x) = x^2 - x + 1,\ x \geq \frac{1}{2}$, then the solution of the equation $f(x) = f^{-1}(x)$ is
(a) $x = 1$ (b) $x = 2$
(c) $x = 1/2$ (d) None of these

21. Consider the binary operation $*$ on Q defined by $a * b = a + 12b + ab$, where $a, b \in Q$. Find $2 * \frac{1}{3}$.
(a) 20/3 (b) 4 (c) 18 (d) 16/3

22. A Signum Function $f : R \to R$ defined by

$$f(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases} \text{ is}$$

(a) one-one (b) onto
(c) many one (d) none of these

23. The number of binary operations that can be defined on a set of 2 elements is
(a) 8 (b) 4 (c) 16 (d) 64

24. If $f : R \to R$ be a function defined by $f(x) = \frac{x^2 - 8}{x^2 + 2}$, then f is
(a) one-one but not onto
(b) one-one and onto
(c) onto but not one-one
(d) neither one-one nor onto

25. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, then $\cot^{-1}x + \cot^{-1}y$ equals
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) π

26. The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{3\sqrt{3}}$

27. If $\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) + \sin^{-1}\left(\frac{2\beta}{1+\beta^2}\right) = 2\tan^{-1}x$, then $x =$
(a) α/β (b) β/α
(c) $\frac{\alpha+\beta}{1+\alpha\beta}$ (d) $\frac{\alpha+\beta}{1-\alpha\beta}$

28. The domain of $f(x) = 2\cos^{-1}2x + \sin^{-1}x$ is
(a) $[-1, 1]$ (b) $[-2, 2]$
(c) $\left[\frac{-1}{2}, \frac{1}{2}\right]$ (d) None of these.

29. The function $f : R \to R$ given by $f(x) = x^3 - 1$ is
(a) one-one but not onto
(b) onto but not one-one
(c) a bijection
(d) neither one-one nor onto

30. Let $g(x) = x^2 - 4x - 5$, then
(a) g is one-one on R (b) g is not one-one on R
(c) g is bijective on R (d) None of these

31. The function $f : R \to R$ defined by $f(x) = 3 - 4x$ is
(a) Onto (b) Not onto
(c) Not one-one (d) None of these

32. If R and R' are symmetric relations (not disjoint) on a set A, then the relation $R \cap R'$ is
(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

33. Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x + x^3}{1 + 3x^2}$, then $fog(x)$ equals
(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) None of these

34. If $f(x) = \cos(\ln x)$, then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right] =$
(a) -1 (b) $1/2$ (c) -2 (d) 0

Assertion & Reasoning Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false and Statement-II is true.

35. Consider the sets $A = \{0, 1, \{0, 1\}, 2, 3\}$ and $B = \{0, 1\}$.
Statement-I : B is a subset of A.
Statement-II : B is an element of A.

36. Statement-I : If set A contains 7 elements and set B contains 6 elements, then the number of one-one onto mapping from A to B is 420.
Statement-II : If A and B are two non-empty sets containing m and n elements respectively, then number of one-one onto functions from A to B = $\begin{cases} n!, \text{ if } m=n \\ 0, \text{ if } m \neq n \end{cases}$.

37. Statement-I : $\sin^{-1}[\tan\{\tan^{-1}x + \tan^{-1}(1-x)\}] = \pi/2$ has no non-zero integral solution.
Statement-II : The maximum and minimum values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are $\frac{7\pi^3}{8}$ and $\frac{\pi^3}{32}$, respectively.

38. Statement-I : Range of $\sin^{-1}x + \tan^{-1}x + \cos^{-1}x$ is $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.
Statement-II : The value of $\sin^{-1}(\cos(4095°))$ is $\frac{\pi}{4}$.

Matrix Match

39. Match the column-I with column-II and select the correct answer using the options given below :

	Column I		Column II
(P)	$\cos(\tan^{-1}x) =$	(1)	x
(Q)	$\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] =$	(2)	$\sqrt{1-x^2}$
(R)	$\sin[\cot^{-1}\{\tan(\cos^{-1}x)\}] =$	(3)	$\frac{1}{\sqrt{1+x^2}}$
(S)	$\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\} =$	(4)	$\sqrt{\frac{x^2+1}{x^2+2}}$

	P	Q	R	S
(a)	1	2	4	3
(b)	3	4	2	1
(c)	4	3	2	1
(d)	3	4	1	2

40. Match column-I with column-II and select the correct answer using the given options.

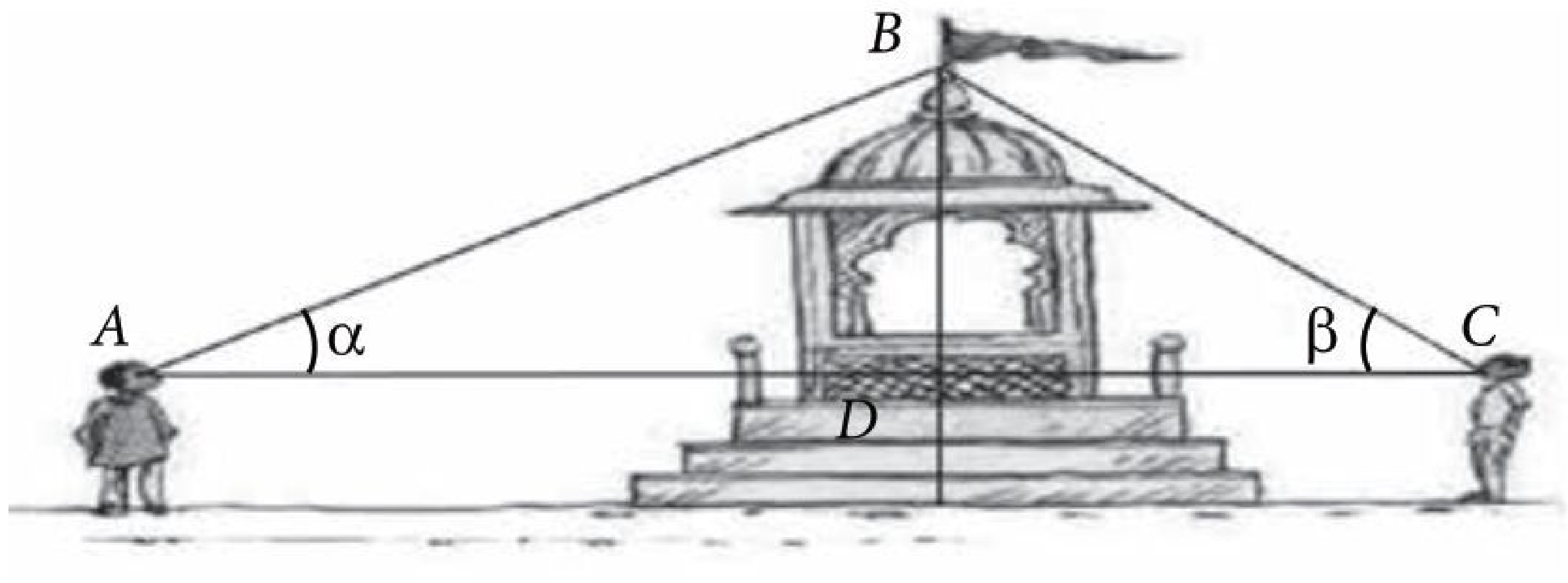
	Column I		Column II
(P)	The number of bijective functions from set A to itself, when A contain 7 elements is	(1)	720
(Q)	If set A has 5 elements and set B has 6 elements, then number of injections that can be formed from A to B is	(2)	5040
(R)	It set A and B contain 4 distinct elements, then the total number of distinct functions from A to B is	(3)	256
(S)	The total number of onto functions from the set $\{1,2, 3, 4\}$ to the set $\{3, 4, 7\}$ is	(4)	36

	P	Q	R	S
(a)	3	4	2	1
(b)	4	3	1	2
(c)	2	1	3	4
(d)	1	2	4	3

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

Two men on either side of a temple, which is 30 metres high above the stairs, observe its top at the angles of elevation α and β respectively (as shown in the figure below). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ metres.



Based on the above information, answer the following questions.

41. $\angle CAB = \alpha =$

(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$
(c) $\sin^{-1}(2)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

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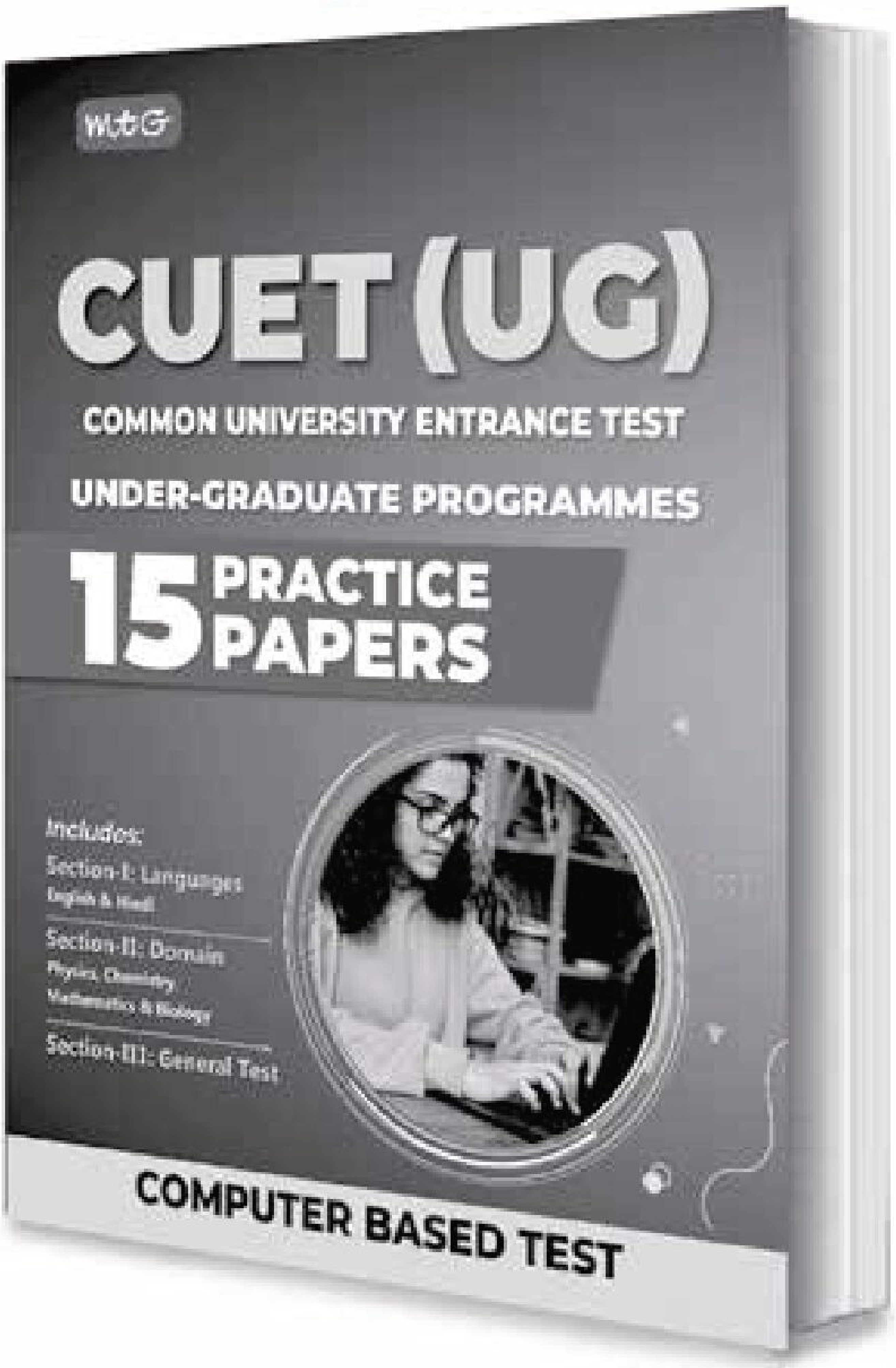
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42. $\angle CAB = \alpha =$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{4}{5}\right)$

43. $\angle BCA = \beta =$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\sqrt{3}\right)$

44. $\angle ABC =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

45. Domain and range of $\cos^{-1} x$ are respectively

(a) $(-1, 1), (0, \pi)$ (b) $[-1, 1], (0, \pi)$

(c) $[-1, 1], [0, \pi]$ (d) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Case II : Read the following passage and answer the questions from 46 to 50.

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1, 2, 3, 4, 5, 6\}$

Based on the above information, attempt any 4 questions.

46. Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : y$ is divisible by $x\}$ is

(a) Reflexive and transitive but not symmetric

(b) Reflexive and symmetric but not transitive

(c) Not reflexive but symmetric and transitive

(d) Equivalence

47. Raji wants to know the number of functions from A to B. How many number of functions are possible?

(a) 6^2 (b) 2^6 (c) $6!$ (d) 2^{12}

48. Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then, R is

(a) Symmetric (b) Reflexive

(c) Transitive (d) None of these

49. Raji wants to know the number of relations possible from A to B. How many number of relations are possible?

(a) 6^2 (b) 2^6 (c) $6!$ (d) 2^{12}

50. Let $R : B \rightarrow B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$, then R is

(a) Symmetric

(b) Reflexive and Transitive

(c) Transitive and symmetric

(d) Equivalence

SOLUTIONS

1. **(c) :** We have, $y = 2x^4$

$\therefore$ $y(-1) = y(1) = 2, y(0) = 0$ (many-one onto)

Here, we see that for two different values of x, we get the same image and no element of y is left, which does not have pre-image. So, function is many-one onto.

2. **(b) :** If $y = \sec^{-1}x$, then $0 \le y \le \pi, y \ne \frac{\pi}{2}$

i.e., $0 \le y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y \le \pi$

3. **(a) :** The number of commutative binary operation on a set of n elements is $n^{\frac{n(n+1)}{2}}$. Required no. of operations $= (2)^{2(2+1)/2} = 8$

4. **(a) :** Let $y = \cos^{-1}(2x - 1) \Rightarrow \cos y = 2x - 1$

Now, $-1 \le 2x - 1 \le 1 \Rightarrow 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$

Hence, the domain of $\cos^{-1}(2x - 1)$ is $[0, 1]$.

5. **(a) :** $ho(gof)(x) = hog(x)^2 = h(\tan x^2) = \log(\tan x^2)$

$$= \log\left[\tan\left(\frac{\sqrt{\pi}}{2}\right)^2\right] = \log(1) = 0$$

6. **(a) :** Domain of $\cos x$ is R and domain of $\sin^{-1} x$ is $[-1, 1]$. Therefore, the domain of $\cos x + \sin^{-1}x$ is $R \cap [-1, 1]$ *i.e.*, $[-1, 1]$.

7. **(a) :** (i) Reflexive : $a \in R$, $aR_1a \Rightarrow |a| = |a|$

(ii) Symmetric : $a, b \in R$

$aR_1b \Rightarrow |a| = |b| \Rightarrow |b| = |a| \Rightarrow bR_1a$

(iii) Transitive : $a, b, c \in R$

$aR_1b \Rightarrow |a| = |b|, bR_1c \Rightarrow |b| = |c|$. So, $|a| = |c| \Rightarrow aR_1c$

$\Rightarrow$ R_1 is an equivalence relation on R.

8. **(c) :** Consider, $\sin^{-1}[\cos(\sin^{-1} x)] + \cos^{-1} [\sin (\cos^{-1} x)]$

$$= \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \sin^{-1} x\right)\right] + \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \cos^{-1} x\right)\right]$$

$$= \frac{\pi}{2} - \sin^{-1} x + \frac{\pi}{2} - \cos^{-1} x = \pi - (\sin^{-1} x + \cos^{-1} x)$$

$$= \pi - \frac{\pi}{2} = \frac{\pi}{2} \qquad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$$

9. (c) : Given, $f(x)=\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)\ldots\infty}}}$

$=(1-\cos x)^{1/2}\,(1-\cos x)^{1/4}\,(1-\cos x)^{1/8}\ldots\infty$

$=(1-\cos x)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\infty}=(1-\cos x)^{\frac{1/2}{1-1/2}}=1-\cos x$

We have, $-1\le\cos x\le 1$

$\Rightarrow\quad -1\le-\cos x\le 1\Rightarrow 0\le 1-\cos x\le 2$

$\therefore$ Range of $f(x)$ is $[0, 2]$.

10. (c) : $\cot^{-1}(2\cdot 1^2)+\cot^{-1}(2\cdot 2^2)+\cot^{-1}(2\cdot 3^2)+\ldots$ upto ∞

$$=\sum_{n=1}^{\infty}\cot^{-1}\left[\frac{(2n+1)(2n-1)+1}{(2n+1)-(2n-1)}\right]$$

$$=\sum_{n=1}^{\infty}\{\cot^{-1}(2n-1)-\cot^{-1}(2n+1)\}$$

$$=(\cot^{-1}1-\cot^{-1}3)+(\cot^{-1}3-\cot^{-1}5)+(\cot^{-1}5-\cot^{-1}7)+\ldots-\cot^{-1}\infty$$

$$=\cot^{-1}1-\cot^{-1}\infty=\frac{\pi}{4}-0=\frac{\pi}{4}.$$

11. (a) : Let $f(x)=y=\dfrac{2x}{x+1}$

$\Rightarrow\quad 2x=yx+y\Rightarrow x=\dfrac{y}{2-y}$

Now, $x\in[0,\infty)\Rightarrow 0\le y<2$

$\Rightarrow\quad$ Range of $f(x)=[0, 2)$

Since range $\subset$ co-domain

$\Rightarrow\quad f$ is into. Clearly, f is one-one also.

12. (c) : Given, $\sin^{-1}x\cos^{-1}x=p$

Now, $(\sin^{-1}x)^3+(\cos^{-1}x)^3=(\sin^{-1}x+\cos^{-1}x)[(\sin^{-1}x)^2+(\cos^{-1}x)^2-\sin^{-1}x\cos^{-1}x]$

$=\dfrac{\pi}{2}[(\sin^{-1}x+\cos^{-1}x)^2-2\sin^{-1}x\cos^{-1}x-\sin^{-1}x\cos^{-1}x]$

$=\dfrac{\pi}{2}\left[\left(\dfrac{\pi}{2}\right)^2-3p\right]\qquad[\because a^3+b^3=(a+b)(a^2-ab+b^2)]$

$=\dfrac{\pi}{2}\left[\dfrac{\pi^2}{4}-3p\right]$

13. (c) : Given binary operation is $a*b=2a+b-3$

On putting $a=3$ and $b=4$, we get

$3*4=2(3)+4-3=6+4-3=7$

14. (d): Given, R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy\Leftrightarrow x$ is relative prime to y.

$\therefore\ R=\{(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)\}$

$\therefore\quad$ Domain of R is $\{2, 3, 4, 5\}$

15. (b): $\sin^{-1}x+\sin^{-1}y=\pi-\sin^{-1}z$

$\Rightarrow\quad x\sqrt{1-y^2}+y\sqrt{1-x^2}=\sin(\pi-\sin^{-1}z)=z$

$\Rightarrow\quad x\sqrt{1-y^2}=z-y\sqrt{1-x^2}$

Squaring both sides, we get

$x^2(1-y^2)=z^2+y^2(1-x^2)-2zy\sqrt{1-x^2}$

$\Rightarrow\quad x^2-y^2-z^2=-2zy\sqrt{1-x^2}$

Again squaring both sides, we get

$x^4+y^4+z^4-2x^2y^2+2y^2z^2-2x^2z^2=4z^2y^2-4x^2y^2z^2$

$\Rightarrow\quad x^4+y^4+z^4+4x^2y^2z^2=2x^2y^2+2y^2z^2+2z^2x^2$

$=2(x^2y^2+y^2z^2+z^2x^2)$

16. (a) : Let $f: A\rightarrow B$ be a one-one function.

$\therefore\quad$ Since, $n(A)=3=p$ and $n(B)=5=q$

$\therefore\quad$ Total number of one-one functions $={}^{q}P_{p}={}^{5}P_{3}=60$

17. (c) : We have, $4\cos^{-1}x+\sin^{-1}x=\pi$

$\Rightarrow\quad 4\left\{\dfrac{\pi}{2}-\sin^{-1}x\right\}+\sin^{-1}x=\pi$

$\Rightarrow\quad 2\pi-4\sin^{-1}x+\sin^{-1}x=\pi\quad\Rightarrow\quad 3\sin^{-1}x=\pi$

$\Rightarrow\quad \sin^{-1}x=\dfrac{\pi}{3}\quad\Rightarrow\quad x=\sin\dfrac{\pi}{3}=\dfrac{\sqrt{3}}{2}$

18. (d): We have, $f(g(x))=x\Rightarrow f(3^{10}x-1)=x$

$\Rightarrow 2^{10}(3^{10}x-1)+1=x\Rightarrow x(2^{10}3^{10}-1)=2^{10}-1$

$$\Rightarrow\quad x=\frac{2^{10}-1}{2^{10}3^{10}-1}\Rightarrow x=\frac{2^{10}(1-2^{-10})}{2^{10}(3^{10}-2^{-10})}=\frac{1-2^{-10}}{3^{10}-2^{-10}}$$

19. (c) : Let $\cos^{-1}x=y$. Then, $x=\cos y$.

$\therefore\quad 2\sin^{-1}\sqrt{\dfrac{1-x}{2}}=2\sin^{-1}\left\{\sqrt{\dfrac{1-\cos y}{2}}\right\}$

$=2\sin^{-1}\left\{\sin\dfrac{y}{2}\right\}=2\left(\dfrac{y}{2}\right)=y$ and $2\cos^{-1}\sqrt{\dfrac{1+x}{2}}$

$=2\cos^{-1}\left\{\sqrt{\dfrac{1+\cos y}{2}}\right\}=2\cos^{-1}\left\{\cos\dfrac{y}{2}\right\}=2\left(\dfrac{y}{2}\right)=y$

Hence, $\cos^{-1}x=2\sin^{-1}\sqrt{\dfrac{1-x}{2}}=2\cos^{-1}\sqrt{\dfrac{1+x}{2}}$

20. (a) : Let $y=x^2-x+1\Rightarrow x^2-x+(1-y)=0$

$\Rightarrow\quad x=\dfrac{1\pm\sqrt{1-4(1-y)}}{2}=\dfrac{1}{2}\pm\sqrt{y-\dfrac{3}{4}}$

$\Rightarrow\quad f^{-1}(y)=\dfrac{1}{2}\pm\sqrt{y-\dfrac{3}{4}}$

$\Rightarrow\quad f^{-1}(x)=\dfrac{1}{2}+\sqrt{x-\dfrac{3}{4}}\qquad\left(\text{As } x\ge\dfrac{1}{2}\right)$

Now, $f(x)=f^{-1}(x)\ \Rightarrow\ x^2-x+1=\dfrac{1}{2}+\sqrt{x-\dfrac{3}{4}}$

$\Rightarrow\quad x=1$ [By checking all possibilities in given options]

21. (a) : We have, $a * b = a + 12\,b + ab\ \forall\ a, b \in Q$

$$\therefore\quad 2*\frac{1}{3}=2+12\left(\frac{1}{3}\right)+2\left(\frac{1}{3}\right)=\frac{20}{3}$$

22. (c) : We have,

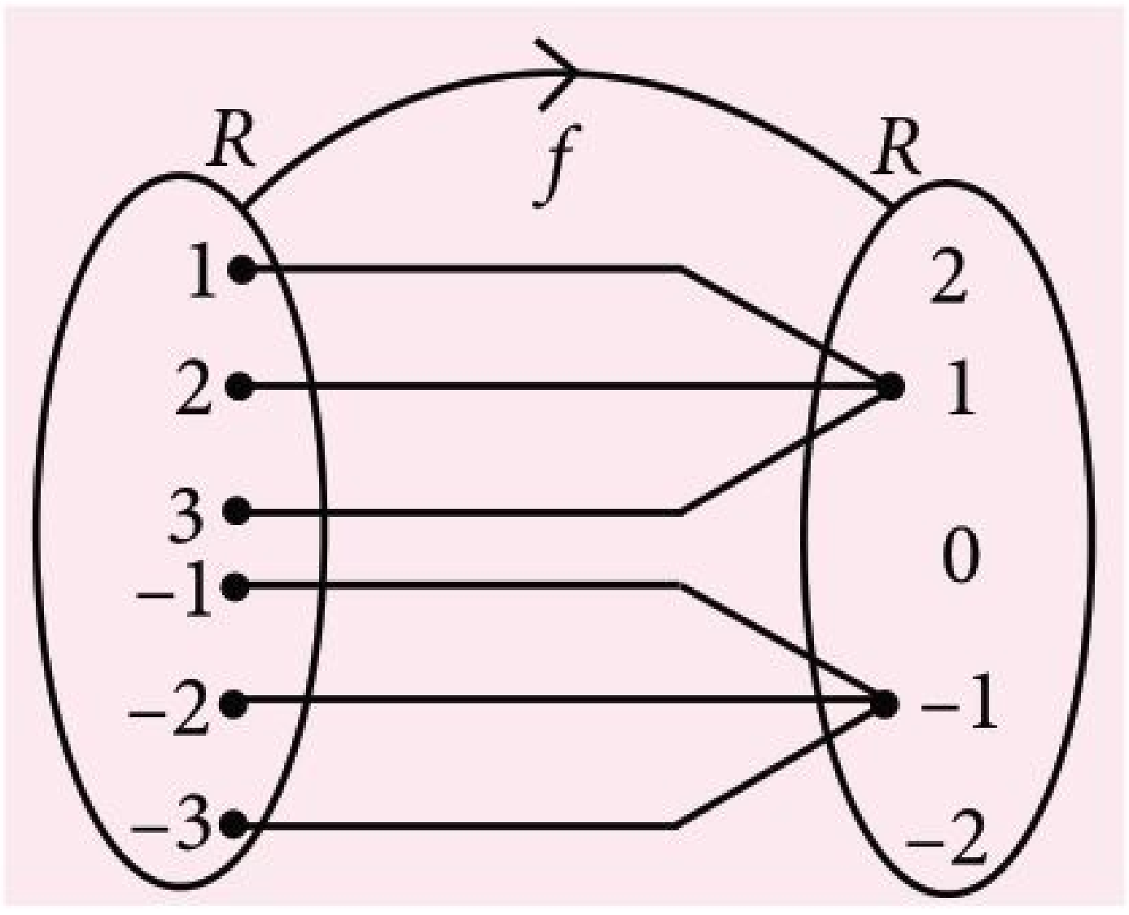
$f(1) = f(2) = f(3) = 1$

$f(0) = 0$ and $f(-1)$

$= f(-2) = f(-3) = -1$

Since, f is not one-one.

$\therefore$ Signum Function is many one.



23. (c) : The number of binary operations on a set consisting of n elements $= n^{n^2}$

$\therefore$ Required no. of binary operations $= 2^{2^2} = 16$

24. (d) : Given function $f : R \to R$ defined by

$$f(x)=\frac{x^2-8}{x^2+2}$$

We observe that for negative and positive values of x, we get same value of $f(x)$.

Hence, $f(x)$ is not one-one.

Also, $y = f(x) \Rightarrow y=\dfrac{x^2-8}{x^2+2} \Rightarrow yx^2+2y=x^2-8$

$$\Rightarrow\ x=\sqrt{\frac{2y+8}{1-y}}$$

For $y = 1$, we can't define x, hence f is not onto.

25. (a) : $\tan^{-1}x + \tan^{-1}y = \dfrac{4\pi}{5}$

$$\Rightarrow\ \frac{\pi}{2}-\cot^{-1}x+\frac{\pi}{2}-\cot^{-1}y=\frac{4\pi}{5}$$

$$\Rightarrow\ \pi-\frac{4\pi}{5}=\cot^{-1}x+\cot^{-1}y \Rightarrow \cot^{-1}x+\cot^{-1}y=\frac{\pi}{5}$$

26. (c) : Consider, $\sin\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\dfrac{\sqrt{63}}{8} = x \Rightarrow \sin x = \dfrac{\sqrt{63}}{8} \Rightarrow \cos x = \dfrac{1}{8}$

Now, $\cos\dfrac{x}{2}=\sqrt{\dfrac{1+\cos x}{2}}=\sqrt{\dfrac{1+1/8}{2}}=\dfrac{3}{4}$

$$\therefore\quad \sin\frac{x}{4}=\sqrt{\frac{1-\cos\frac{x}{2}}{2}}=\sqrt{\frac{1-\frac{3}{4}}{2}}=\sqrt{\frac{1}{8}}=\frac{1}{2\sqrt{2}}$$

27. (d) : $\sin^{-1}\left(\dfrac{2\alpha}{1+\alpha^2}\right)+\sin^{-1}\left(\dfrac{2\beta}{1+\beta^2}\right)=2\tan^{-1}x$

$\Rightarrow\ 2\tan^{-1}\alpha + 2\tan^{-1}\beta = 2\tan^{-1}x$

$\Rightarrow\ \tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}x$

$$\Rightarrow\ \tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)=\tan^{-1}x \ \Rightarrow\ x=\frac{\alpha+\beta}{1-\alpha\beta}$$

28. (c) : The domain of $\cos^{-1}x$ is $[-1, 1]$ and $\sin^{-1}x$ is $[-1, 1]$.

$\therefore$ $\cos^{-1}2x$ is defined for all x satisfying $-1 \le 2x \le 1$

$$\Rightarrow\ \frac{-1}{2}\le x\le\frac{1}{2}$$

Thus, domain of $2\cos^{-1}2x + \sin^{-1}x$ is $\left[\dfrac{-1}{2},\dfrac{1}{2}\right]$.

29. (c) : Given, $f(x) = x^3 - 1$. Let $x_1, x_2 \in R$.

Now, $f(x_1) = f(x_2) \Rightarrow x_1^3 - 1 = x_2^3 - 1 \Rightarrow x_1 = x_2$

$\therefore$ $f(x)$ is one-one. Also, it is onto.

Hence, it is a bijection.

30. (b) : Let $g(x_1) = g(x_2) \Rightarrow x_1^2-4x_1-5=x_2^2-4x_2-5$

$\Rightarrow x_1^2-x_2^2=4(x_1-x_2) \Rightarrow (x_1-x_2)(x_1+x_2-4)=0$

Either $x_1 = x_2$ or $x_1 + x_2 = 4$. Either $x_1 = x_2$ or $x_1 = 4 - x_2$

$\therefore$ There are two values of x_1, for which $g(x_1) = g(x_2)$.

$\therefore$ $g(x)$ is not one-one $\forall\ x \in R$

31. (a) : Let $y \in R$ be any real number, such that $f(x) = y$

$\therefore\ y = 3 - 4x \Rightarrow 4x = 3 - y \Rightarrow x = \dfrac{3-y}{4}$

So, for any real number $y \in R$, there exists $\dfrac{3-y}{4} \in R$ such that $f\left(\dfrac{3-y}{4}\right)=3-4\left(\dfrac{3-y}{4}\right)=3-3+y=y$

Hence, f is onto.

32. (b) : Given R *and* R' are not disjoint, so there is atleast one ordered pair, say, $(a, b) \in R \cap R'$.

$\Rightarrow\ (a, b) \in R$ and $(a, b) \in R'$.

As R and R' are symmetric relations, we get

$(b, a) \in R$ and $(b, a) \in R' \Rightarrow (b, a) \in R \cap R'$

Hence, $R \cap R'$ is symmetric.

33. (b) : $f[g(x)]=f\left[\dfrac{3x+x^3}{1+3x^2}\right]=\log\left\{\dfrac{1+\dfrac{3x+x^3}{1+3x^2}}{1-\dfrac{3x+x^3}{1+3x^2}}\right\}$

$$\Rightarrow\ f[g(x)]=\log\left(\frac{1+x}{1-x}\right)^3=3\log\left(\frac{1+x}{1-x}\right)=3f(x)$$

34. (d) : $f(xy) = \cos\ln(xy) = \cos(\ln x + \ln y)$

Also, $f\left(\dfrac{x}{y}\right)=\cos\ln\left(\dfrac{x}{y}\right)=\cos(\ln x-\ln y)$

Now, $f(xy)+f\left(\dfrac{x}{y}\right)=\cos(\ln x+\ln y)+\cos(\ln x-\ln y)$

$= 2\cos\ln x \cdot \cos\ln y = 2f(x)\,f(y)$

$$\therefore\quad f(x)\,f(y)-\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]=0$$

35. (b) : Here, both the statements are true as $0, 1 \in A$, therefore, $B \subset A$ and also $B = \{0, 1\} \in A$.

36. **(d):** Clearly, Statement-II is true.
Now, $m = 7$ and $n = 6$ *i.e.*, $m \neq n$

$\therefore$ Number of one-one onto mapping from A to B is 0.
Statement-I is false and Statement-II is true.

37. **(d):** $\sin^{-1}[\tan\{\tan^{-1}x + \tan^{-1}(1 - x)\}] = \pi/2$

$$\Rightarrow \sin^{-1}\left[\tan\left\{\tan^{-1}\left(\frac{x+1-x}{1-x(1-x)}\right)\right\}\right] = \pi/2$$

$$\Rightarrow \tan\left(\tan^{-1}\left(\frac{1}{1-x+x^2}\right)\right) = \sin\frac{\pi}{2} = 1$$

$$\Rightarrow \frac{1}{1-x+x^2} = 1 \Rightarrow x^2 + 1 - x = 1$$

$$\Rightarrow x(x - 1) = 0 \Rightarrow x = 0, 1$$

$\therefore$ Statement-I is false.

Let $y = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$.

Then, $y = (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x)$

$$\Rightarrow y = \left(\frac{\pi}{2}\right)^3 - \frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$\Rightarrow y = \frac{\pi^3}{8} - \frac{3\pi^2}{4}\sin^{-1}x + \frac{3\pi}{2}(\sin^{-1}x)^2$$

$$\Rightarrow \frac{3\pi}{2}(\sin^{-1}x)^2 - \frac{3\pi^2}{4}(\sin^{-1}x) + \left(\frac{\pi^3}{8} - y\right) = 0$$

$$\Rightarrow (\sin^{-1}x)^2 - \frac{\pi}{2}(\sin^{-1}x) + \frac{2}{3\pi}\left(\frac{\pi^3}{8} - y\right) = 0$$

$$\Rightarrow \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16} + \frac{\pi^2}{12} - \frac{2y}{3\pi} = 0$$

$$\Rightarrow \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{48} - \frac{2y}{3\pi} = 0$$

$$\Rightarrow \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 = \frac{2y}{3\pi} - \frac{\pi^2}{48} \quad \ldots\text{(i)}$$

We know that $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$, for all $x \in [-1,1]$

$$\Rightarrow -\frac{3\pi}{4} \le \sin^{-1}x - \frac{\pi}{4} \le \frac{\pi}{4}$$

$$\Rightarrow 0 \le \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16} \quad \ldots\text{(ii)}$$

From (i) and (ii), we find that

$$0 \le \frac{2y}{3\pi} - \frac{\pi^2}{48} \le \frac{9\pi^2}{16}$$

$$\Rightarrow \frac{\pi^2}{48} \le \frac{2y}{3\pi} \le \frac{9\pi^2}{16} + \frac{\pi^2}{48} \Rightarrow \frac{\pi^3}{32} \le y \le \frac{7\pi^3}{8}$$

Hence, the minimum and maximum values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are $\frac{\pi^3}{32}$ and $\frac{7\pi^3}{8}$, respectively.

$\therefore$ Statement-I is false, Statement-II is true.

38. **(c) :** Consider, $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$.
Clearly, domain of the function is $[-1, 1]$

Also, for $[-1, 1]$, $\tan^{-1}x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

Now, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, for $x \in [-1, 1]$

Thus, $f(x) = \tan^{-1}x + \frac{\pi}{2}$, where $x \in [-1, 1]$

Hence, the range is $\left[\frac{-\pi}{4} + \frac{\pi}{2}, \frac{\pi}{4} + \frac{\pi}{2}\right] = \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$\therefore$ Statement-I is true.

Now, $\sin^{-1}[\cos(4095°)] = \sin^{-1}\left(\cos\left(\frac{45\pi}{2} + \frac{\pi}{4}\right)\right)$

Since, $\cos\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n+1}{2}}\sin\theta$

$$\therefore \sin^{-1}[\cos(4095°)] = \sin^{-1}\left((-1)^{\frac{45+1}{2}}\sin(\pi/4)\right)$$

$$= \sin^{-1}(-\sin(\pi/4)) = \sin^{-1}(\sin(-\pi/4)) = -\frac{\pi}{4}$$

$\therefore$ Statement-I is true but Statement-II is false.

39. **(d):** **(P)** Consider, $\cos(\tan^{-1}x)$

$$= \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$$

$$\left[\because \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$$

(Q) Consider, $\sin(\cot^{-1}x) = \sin\left\{\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right\} = \frac{1}{\sqrt{1+x^2}}$

$\therefore$ $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

$$= \cos\left\{\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right\} = \cos\left\{\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right\} = \sqrt{\frac{x^2+1}{x^2+2}}$$

(R) Consider, $\sin[\cot^{-1}\{\tan(\cos^{-1}x)\}]$

$$= \sin\left[\cot^{-1}\left\{\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)\right\}\right]$$

$$= \sin\left[\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right] = \sin\left[\sin^{-1}\left(\frac{x}{1}\right)\right] = x$$

(S) Consider, $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) \quad \ldots\text{(i)}$

$$= \sin\left(2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right) \quad \text{(By putting } x = \cos\theta \text{ in (i))}$$

$$= \sin\left(2\tan^{-1}\left(\sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}\right)\right)$$

$$= \sin\left(2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right) = \sin\left(2\times\frac{\theta}{2}\right)$$

$$= \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-x^2}$$

40. (c) : (P) We know that if A and B are two sets consisting of m and n elements, respectively, then the number of bijections that can be formed are $m!$, if $m = n$.

$\therefore$ Total number of bijective functions from set A to itself $= 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

(Q) We know that if A and B are two sets consisting of m and n elements, respectively, then the total number of injections that can be formed are ${}^nP_m$, $n \geq m$.

Here, $m = 5$ and $n = 6$

$\therefore$ Total number of one one functions from

$$A \text{ to } B = {}^6P_5 = \frac{6!}{(6-5)!} = \frac{6\times5\times4\times3\times2\times1}{1!} = 720$$

(R) We know that if A and B are two sets consisting of m and n elements, respectively, then the total number of functions from A to B is n^m.

Here, $n = m = 4$

$\therefore$ Total number of functions $= 4^4 = 256$

(S) Since, we know that if A and B are two sets consisting of m and n elements, respectively such that $1 \leq n \leq m$, then the number of onto functions from A to B is

$$\sum_{r=1}^{n}(-1)^{n-r}\,{}^nC_r r^m \text{ if } m \geq n$$

Here, $m = 4$, $n = 3$

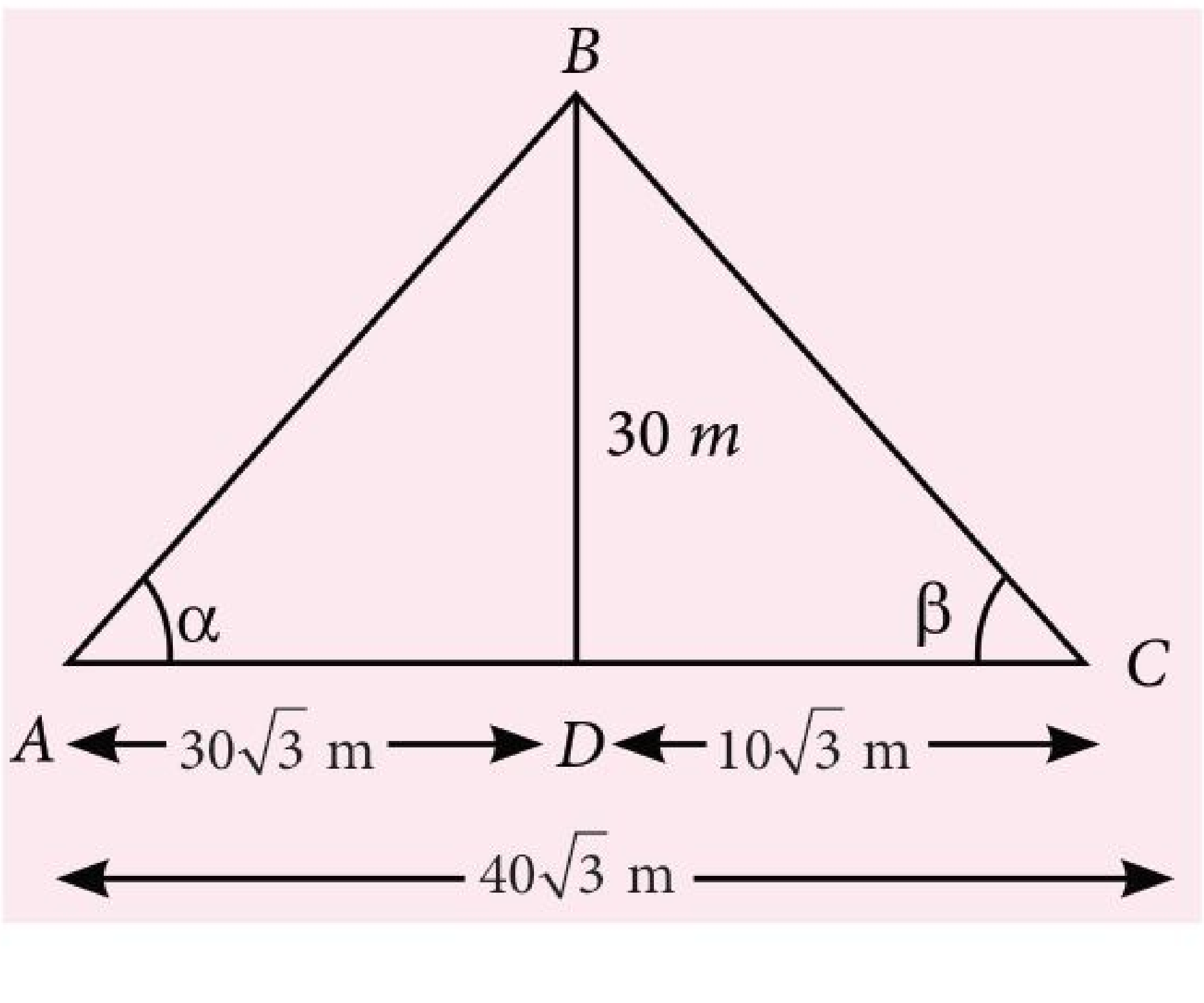
$\therefore$ Total number of onto functions

$$= \sum_{r=1}^{3}(-1)^{3-r}\,{}^3C_r\, r^4 = {}^3C_1 - {}^3C_2\cdot 2^4 + {}^3C_3\cdot 3^4$$

$= 3 - 48 + 81 = 36$

41. (b) : We have, $BD = 30$ m,

$AC = 40\sqrt{3}$ m and $AD = 30\sqrt{3}$ m



Clearly, $BD \perp AC$

$\therefore$ In right ΔADB, $AB^2 = AD^2 + BD^2$

$= (30\sqrt{3})^2 + (30)^2 = 3600 \quad \Rightarrow AB = 60$ m

Now, $\sin\alpha = \frac{DB}{AB} = \frac{30}{60} = \frac{1}{2}$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \angle CAB = \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

42. (c) : In right ΔADB, $\cos\alpha = \frac{AD}{AB} = \frac{30\sqrt{3}}{60} = \frac{\sqrt{3}}{2}$

$$\Rightarrow \angle CAB = \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

43. (d) : Clearly, $CD = AC - AD$

$= 40\sqrt{3} - 30\sqrt{3} = 10\sqrt{3}$ m

$$\therefore \quad \tan\beta = \frac{BD}{CD} = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \angle BCA = \beta = \tan^{-1}(\sqrt{3})$$

44. (c) : $\sin\alpha = \frac{1}{2} \Rightarrow \alpha = 30^\circ$

$\tan\beta = \sqrt{3} \Rightarrow \beta = 60^\circ$

$\therefore \angle ABC = 180^\circ - (\alpha+\beta) = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$

or $\quad \angle ABC = \frac{\pi}{2}$

45. (c) : Domain and range of $\cos^{-1}x$ are $[-1, 1]$ and $[0, \pi]$, respectively.

46. (a) : $R = \{(x, y) : y \text{ is divisible by } x\}$

$= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (1, 3), (1, 4), (2, 4), (1, 5), (1, 6), (2, 6), (3, 6)\}$

Here, $(x, x) \in R\ \forall\ x \in B \Rightarrow R$ is reflexive

$(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric

Clearly, R is transitive also.

47. (a) : Here, $n(A) = 2$ and $n(B) = 6$

$\therefore$ Total no. of functions from A to $B = 6 \times 6 = 6^2$

48. (d) : R is not reflexive as $(1, 1), (3, 3), (4, 4), (6, 6) \notin R$.

R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$.

R is not transitive as $(1, 3) \in R$ and $(3, 4) \in R$ but $(1, 4) \notin R$.

49. (d) : We have, $n(A) = 2$, $n(B) = 6$

No. of relations from A to $B = 2^{n(A)\times n(B)} = 2^{2\times 6} = 2^{12}$

50. (b) : $(x, x) \in R\ \forall\ x \in B$

$\therefore$ R is reflexive.

$(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric

Also, R is transitive.

◆◆

beat the TIME TRAP

Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. If the sides of a triangle are 8 cm, 10 cm and 12 cm, then the relation between the greatest angle and the smallest angle is, (where C is greatest angle and B is smallest angle)

(a) $A = C$ (b) $C = 2A$
(c) $C = 3A$ (d) $A = 2C$

2. If $1 + b + b^2 + ... + b^n = (1 + b)(1 + b^2)(1 + b^4)(1 + b^8)$, then n is given by

(a) 7 (b) 9 (c) 15 (d) 17

3. The straight line $4x + 3y = p$ is tangent to the circle $x^2 + y^2 + 4\sqrt{3} - 4x - 6y = 0$, then p equals

(a) $10 + 12\sqrt{3}$ (b) $10 - 22\sqrt{3}$
(c) $12 + 10\sqrt{3}$ or $22 - 10\sqrt{3}$
(d) $22 + 10\sqrt{3}$ or $12 - 10\sqrt{3}$

4. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy \cos\alpha + y^2$ is equal to

(a) $4 \sin^2\alpha$ (b) $2 \sin^2\alpha$
(c) $4\sin^2\alpha - 2x^2y^2$ (d) $4\cos^2\alpha + 2x^2y^2$

5. Let z and ω be two complex numbers such that $|z| \le 1$, $|\omega| \le 1$, then $(|z| - |\omega|)^2 + (\arg z - \arg\omega)^2 \ge$

(a) $|z - \omega|^2$ (b) $|z + \omega + 1|^2$
(c) $|z - \omega - 1|^2$ (d) None of these

6. If $\int \frac{dx}{4\sin^2 x + 4\sin x \cos x + 5\cos^2 x}$ $= A \tan^{-1}(B \tan x + C)$, then

(a) $A = 1/4, B = 1/2, C = 1$
(b) $A = \frac{1}{2}, B = \frac{1}{4}, C = 1$
(c) $A = 1, B = 1/2, C = 1/4$
(d) $A = 1/4, B = 1, C = 1/2$

7. The value of $\int_{-2}^{0} [x^3 + 3x^2 + 3x + 3x + 3 + (x + 1)$ $\cos (x + 1)]\, dx$ is equal to

(a) 0 (b) 3 (c) 4 (d) 1

8. The area bounded by the curve $y = \frac{1}{1+\sqrt{\tan x}}$ and x-axis between the ordinates $x = \pi/6$ and $x = \pi/3$ is

(a) $\frac{\pi}{4}$ sq. units (b) $\frac{\pi}{2}$ sq. units
(c) $\frac{\pi}{5}$ sq. units (d) $\frac{\pi}{12}$ sq. units

9. Chord AB is a diameter of the sphere $|\vec{r} - 2\hat{i} - \hat{j} + 6\hat{k}| = \sqrt{18}$ with the coordinates of A as $(3, 2, -2)$, then coordinates of B equals

(a) $(1, 0, 10)$ (b) $(-1, 0, -10)$
(c) $(-3, -2, -2)$ (d) $(1, 0, -10)$

10. If a wire of length a units is cut into three pieces, the n probability that three pieces form a triangle is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

SECTION-II

Numerical Answer Type

11. If $z = \frac{\sqrt{3} - i}{2}$ and $(z^{95} + i^{71})^{94} = z^n$, then the sum of digits of the smallest positive integral values of n equals ______ .

12. The number of ways in which 13 identical gold coins can be distributed among three persons such that each person can get at least two gold coins is ______ .

13. If pair of lines $x^2 - 4pxy - y^2 = 0$ and $x^2 + 6qxy - y^2 = 0$ is such that each pair bisects the angle between the other pair, then $(pq)^{-1}$ equals ______.

14. Let $f(x)=\begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If $f(x)$ be an odd function and its odd values is equal to $g(x)$, then find the absolute value of λ given that $\lambda f(1)g(1) = 4$.

15. Let $f: R \to R$ be a function defined as

$$f(x)=\begin{cases} 3\left(1-\dfrac{|x|}{2}\right), & \text{if } |x|\le 2 \\ 0 \quad , & \text{if } |x|>2 \end{cases}.$$

Let $g : R \to R$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in R, where g is not continuous and not differentiable, respectively, then $n + m$ is equal to ______.

SOLUTIONS

1. (b) : Let $a = 8$ cm, $b = 10$ cm and $c = 12$ cm. Clearly, greatest angle is C and the smallest angle is A, (as we know greatest angle is opposite to greatest side and smallest angle is opposite to smallest side). Here, we have to prove $C = 2A$. Applying cosine rule, we get

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{64+100-144}{2\cdot 8\cdot 10} = \frac{1}{8} \qquad \text{...(i)}$$

$$\text{and } \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{100+144-64}{2\cdot 10\cdot 12} = \frac{3}{4} \qquad \text{...(ii)}$$

Now, $\cos 2A = 2\cos^2 A - 1 = 2 \times \dfrac{9}{16} - 1$ {using(ii)}

$\therefore \ \cos 2A = \dfrac{1}{8}$ ...(iii)

From (i) and (iii), we get

$\cos 2A = \cos C \Rightarrow C = 2A$

2. (c) : Given,

$(1 + b + b^2 + ... + b^n) = (1 + b)(1 + b^2)(1 + b^4)(1 + b^8)$

$\Rightarrow (1 - b^{n+1}) = (1 - b^2)(1 + b^2)(1 + b^4)(1 + b^8)$

$\Rightarrow 1 - b^{n+1} = 1 - b^{16} \Rightarrow n = 15$

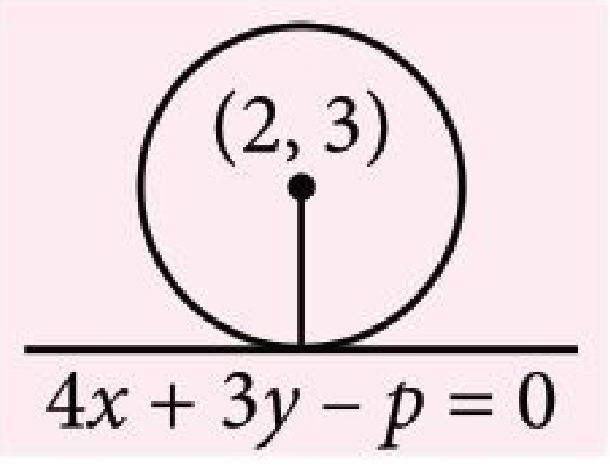
3. (c) : Given $x^2 + y^2 - 4x - 6y + 4\sqrt{3} = 0$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = 13 - 4\sqrt{3}$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = \left(2\sqrt{3}-1\right)^2$

$\Rightarrow$ centre (2, 3) and $R = 2\sqrt{3}-1$

Now $\dfrac{4\cdot 2+3\cdot 3-p}{5} = \pm\left(2\sqrt{3}-1\right)$



$\Rightarrow 17 - p = \pm\,(10\sqrt{3}-5) \Rightarrow 17 \pm (10\sqrt{3}-5) = p$

$\Rightarrow p = 17 + 10\sqrt{3} - 5$ or $p = 17 - 10\sqrt{3} + 5$

$\Rightarrow p = 12 + 10\sqrt{3}$ or $p = 22 - 10\sqrt{3}$

4. (a) : Given, $\cos^{-1} x - \cos^{-1}\dfrac{y}{2} = \alpha$

$$\Rightarrow \cos^{-1}\left\{\frac{xy}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}\right\} = \alpha$$

$$\Rightarrow \frac{xy}{2}+\frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2} = \cos\alpha$$

$$\Rightarrow xy+\sqrt{1-x^2}\sqrt{4-y^2} = 2\cos\alpha$$

$$\Rightarrow \sqrt{1-x^2}\sqrt{4-y^2} = 2\cos\alpha - xy$$

Squaring both sides, we get

$(1 - x^2)(4 - y^2) = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2 = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4 - 4\cos^2\alpha$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4(1 - \cos^2\alpha)$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

5. (a) : Let $z = r_1(\cos\theta + i\sin\theta)$, $\omega = r_2(\cos\alpha + i\sin\alpha)$

$$\begin{aligned}|z-\omega|^2 &= (r_1-r_2)^2 - 2r_1r_2\cos(\theta-\alpha) + 2r_1r_2 \\ &= (r_1-r_2)^2 + 2r_1r_2(1-\cos(\theta-\alpha)) \\ &= (r_1-r_2)^2 + 2r_1r_2 2\sin^2\left(\frac{\theta-\alpha}{2}\right) \\ &\le (r_1-r_2)^2 + 4(1)(1)\left(\frac{\theta-\alpha}{2}\right)^2\end{aligned}$$

$$\left[\because \ \sin^2\left(\frac{\theta-\alpha}{2}\right) \le \left(\frac{\theta-\alpha}{2}\right)^2\right]$$

$\therefore \ |z-\omega|^2 \le (|z|-|\omega|)^2 + (\operatorname{Arg} z - \operatorname{Arg}\omega)^2$

6. (d) : $\displaystyle\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$

$= \displaystyle\int \frac{\sec^2 x\, dx}{4\tan^2 x + 4\tan x + 5}$ (Dividing N^r and D^r by $\cos^2 x$)

$= \displaystyle\int \frac{dt}{(2t+1)^2 + 2^2}$, where $t = \tan x$, $dt = \sec^2 x\, dx$

$= \dfrac{1}{2}\cdot\dfrac{1}{2}\tan^{-1}\left(\dfrac{2t+1}{2}\right) = \dfrac{1}{4}\tan^{-1}(t + 1/2)$

$= \frac{1}{4}\tan^{-1}(\tan x + 1/2) = A\tan^{-1}(B\tan x + C)$

$\therefore\ A = \frac{1}{4},\ B = 1,\ C = 1/2$

7. (c) : Let $I = \int_{-2}^{0}[x^3+3x^2+3x+3+(x+1)\cos(x+1)]dx$

$$= \int_{-2}^{0}[(x+1)^3+2+(x+1)\cos(x+1)]dx$$

Put $x + 1 = t \Rightarrow dx = dt$

$$I = \int_{-1}^{1}(t^3+2+t\cos t)dt = \int_{-1}^{1}t^3\,dt + 2\int_{-1}^{1}dt + \int_{-1}^{1}t\cos t\,dt$$

$= 0 + 2[t]_{-1}^{1} + 0$ ($\because$ t^3 and $t\cos t$ are odd functions)
$= 4$

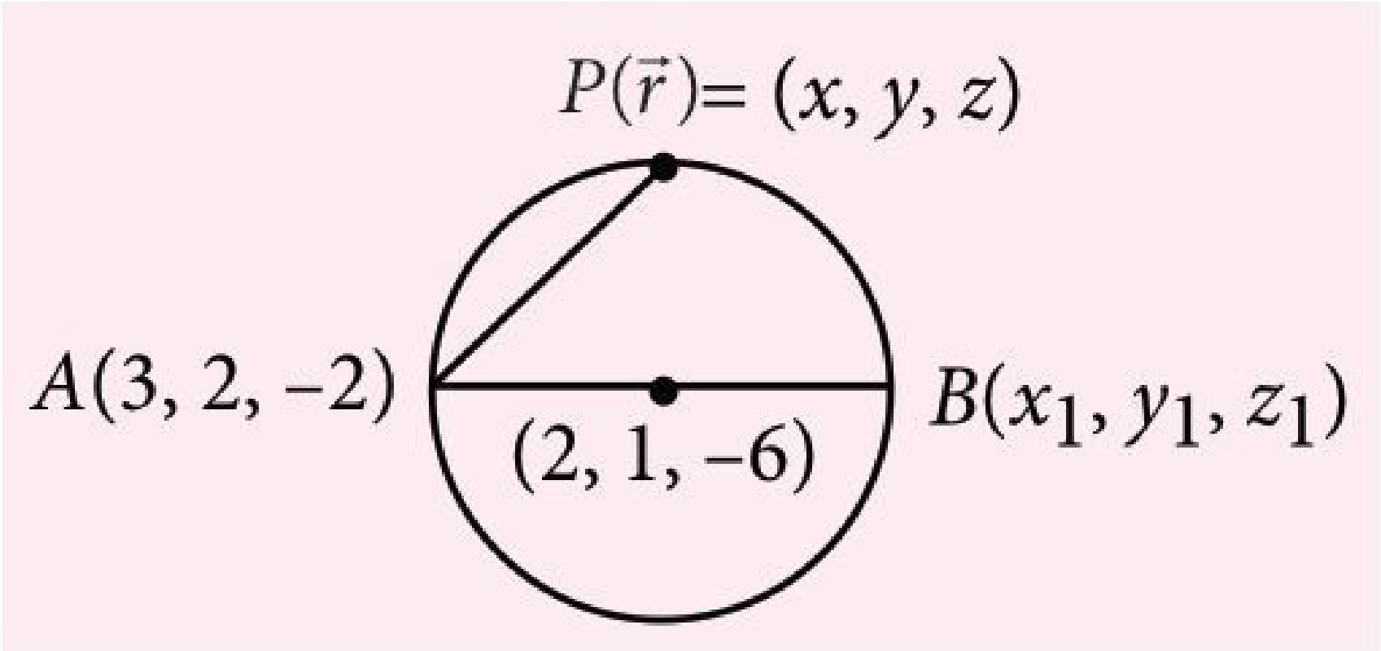
8. (d) : Given curve is $\frac{1}{1+\sqrt{\tan x}} = \frac{\sqrt{\cos x}}{(\sin x)^{1/2}+(\cos x)^{1/2}}$

$\therefore$ Area bounded between the intervals $x = \pi/6$ and $x = \pi/3$ is given by

$$\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \pi/12 \text{ sq. units}$$

9. (d) : $|\vec{r}-\vec{c}| = \sqrt{18}$

$\therefore$ Centre $(2, 1, -6)$



Using mid-point formula,
$x_1 + 3 = 4,\ y_1 + 2 = 2,\ z_1 - 2 = -12$
$\Rightarrow\ x_1 = 1,\ y_1 = 0,\ z_1 = -10$

10. (c) : Let the lengths of three pieces of wire be $x, y, a - (x + y)$, then each part must be > 0.

$\therefore\ x > 0, y > 0, a - (x + y) > 0 \Rightarrow x + y < a$

Now, three parts of wire will form a triangle if the sum of any two parts must be greater than the third part. So, we must have

(i) $x + y > a - (x + y) \Rightarrow 2(x + y) > a \Rightarrow y > \frac{a}{2} - x$

(ii) $x + a - (x + y) > y \Rightarrow a > 2y \Rightarrow y < \frac{a}{2}$

(iii) $y + a - (x + y) > x$

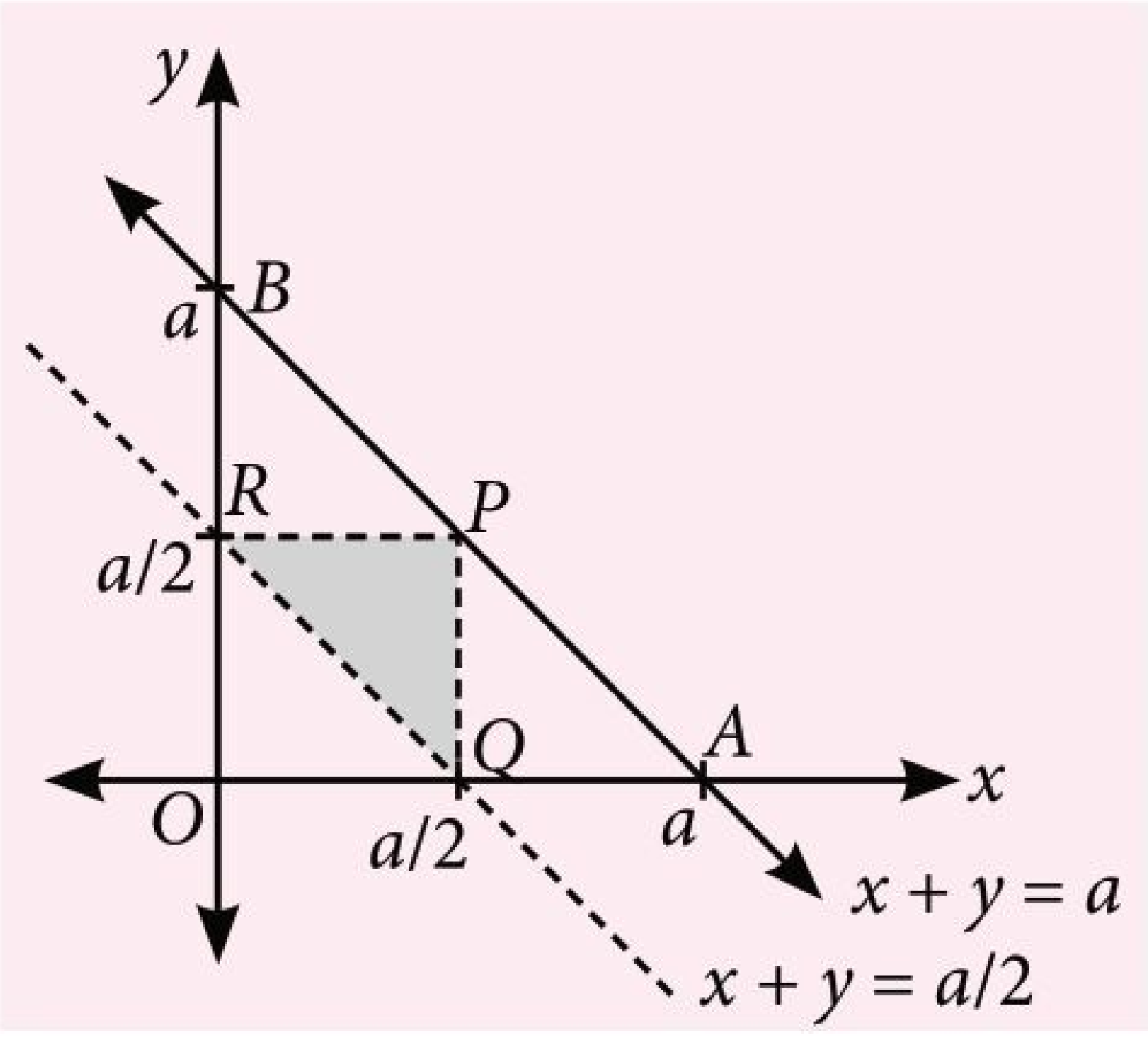
$\Rightarrow\ a > 2x \Rightarrow x < \frac{a}{2}$

From (i), (ii) and (iii),

we have $x + y > \frac{a}{2}$

and $0 < x < \frac{a}{2}$,

$0 < y < \frac{a}{2}$.



$\therefore$ Required probability

$$= \frac{ar(\Delta PQR)}{ar(\Delta AOB)} = \frac{\frac{1}{2}\times\frac{a}{2}\times\frac{a}{2}}{\frac{1}{2}\times a\times a} = \frac{1}{4}$$

11. (10) : Given $z = \frac{\sqrt{3}-i}{2} = i\left(\frac{-1-i\sqrt{3}}{2}\right) = i\omega^2$, where ω is a cube root of unity.

Now, $z^{95} = (i\omega^2)^{95} = i^{4(23)+3}\cdot\omega^{3(63)+1} = -i\omega$
($\because$ $i^{4\lambda} = 1, \omega^{3\lambda} = 1\ \forall\ \lambda = 1, 2, 3, \ldots$)

and $i^{71} = i^{4\lambda+3} = i^3 = -i\ (\lambda \in I)$

$\therefore\ (z^{95} + i^{71})^{94} = (-i\omega + (-i))^{94} = (-i(1+\omega))^{94}$
$= (i\omega^2)^{94} = i^2\omega^2$

Now, $(z^{95} + i^{71})^{94} = z^n = i^2\omega^2$ ...(i)

and $z = i\omega^2\ \therefore\ z^n = i^n\cdot\omega^{2n}$ ...(ii)

$\therefore$ From (i) & (ii), we have $i^2\omega^2 = i^n\omega^{2n}$

or $i^{n-2}\,\omega^{2n-2} = 1$

$\therefore\ n - 2 = 4\lambda_1$ ...(iii)

and $2n - 2 = 3\lambda_2,\ \lambda_1, \lambda_2 \in I$ ...(iv)

On eliminating n from (iii) & (iv), we get

$$2(4\lambda_1+2)-2 = 3\lambda_2 \Rightarrow \lambda_2 = \frac{8\lambda_1+2}{3}$$

$\therefore$ Smallest positive integral values for λ_1 and λ_2 are $\lambda_1 = 2, \lambda_2 = 6$

$\therefore$ Smallest integral value of $n = 4\lambda_1 + 2 = 10$

12. (36) : $x + y + z = 13 - 6 = 7$

$\therefore$ Number of non-negative solutions $= {}^{n+r-1}C_{r-1}$
$= {}^{7+3-1}C_{3-1} = {}^{9}C_2 = 36$.

13. (6) : Equation of angle bisector for $x^2 - 4pxy - y^2 = 0$ is

$$\frac{x^2-y^2}{1-(-1)} = \frac{xy}{-2p} \Rightarrow x^2 + \frac{xy}{p} - y^2 = 0 \quad \text{...(i)}$$

Now, $x^2 + 6qxy - y^2 = 0$ and (i) are identical.

$\therefore\ 6q = \frac{1}{p} \Rightarrow pq = \frac{1}{6}$

14. (1) : $f(-x) = -f(x) = g(x)$

$\Rightarrow f(x)\cdot g(x) = -(f(x))^2$

$$\text{or}\quad f(1)\,g(1) = -(f(1))^2 = -\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}^2 = 4$$

$\Rightarrow \lambda = -1 \Rightarrow |\lambda| = 1$

15. (4) : We have a function $f : R \rightarrow R$ defined as

$$f(x) = \begin{cases} 3\left(1 - \dfrac{|x|}{2}\right), & \text{if } |x| \le 2 \\ 0, & \text{if } |x| > 2 \end{cases}$$

$$f(x+2) = \begin{cases} 3\left(\dfrac{4+x}{2}\right), & \text{if } -4 \le x \le -2 \\ \dfrac{-3x}{2}, & \text{if } -2 < x \le 0 \\ 0, & \text{if } x \in (-\infty, -4) \cup (0, \infty) \end{cases}$$

$$-f(x-2) = \begin{cases} \dfrac{-3x}{2}, & \text{if } 0 \le x \le 2 \\ \dfrac{3(x-4)}{2}, & \text{if } 2 < x \le 4 \\ 0, & \text{if } x \in (-\infty, 0) \cup (4, \infty) \end{cases}$$

Now, $g(x) = f(x + 2) - f(x - 2)$

$$\therefore\ g(x) = \begin{cases} 0+0, & \text{if } x \in (-\infty, -4] \\ \dfrac{3(4+x)}{2}, & \text{if } -4 < x \le -2 \\ \dfrac{-3x}{2}, & \text{if } -2 < x \le 0 \\ \dfrac{-3x}{2}, & \text{if } 0 < x \le 2 \\ \dfrac{3(x-4)}{2}, & \text{if } 2 < x \le 4 \\ 0, & \text{if } x \in (4, \infty) \end{cases}$$

$$\text{and } g'(x) = \begin{cases} 0, & \text{if } x \in (-\infty, -4] \\ \dfrac{3}{2}, & \text{if } -4 < x \le -2 \\ \dfrac{-3}{2}, & \text{if } -2 < x \le 0 \\ \dfrac{-3}{2}, & \text{if } 0 < x \le 2 \\ \dfrac{3}{2}, & \text{if } 2 < x \le 4 \\ 0, & \text{if } x \in (4, \infty) \end{cases}$$

$\therefore\ n = 0$ and $m = 4 \Rightarrow n + m = 4$

◆ ◆

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OLYMPIAD CORNER

1. Prove that for all positive intergers n, $p_n(x) = x^n - 2$ is irreducible in $Q[x]$.
2. Let r be a line in the plane and let ABC be a triangle contained in one of the half-planes determined by r. Let A', B', C', be the points symmetric to A, B, C with respect to r; draw the line through A' parallel to BC, the line through B' parallel to AC and the line through C' parallel to AB. Show that these three lines have a common point.
3. Determine all the possible values of the sum of the digits of the perfect squares.
4. Let x be a real number with $0 < x < \pi$. Prove that, for all natural numbers n, the sum
$$\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots + \frac{\sin(2n-1)x}{2n-1} \text{ is positive.}$$
5. An acute triangle ABC is given. Points A_1 and A_2 are taken on the side BC (with A_2 between A_1 and C), B_1 and B_2 on the side AC (with B_2 between B_1 and A) and C_1 and C_2 on the side AB (with C_2 between C_1 and B) so that
$$\angle AA_1A_2 = \angle AA_2A_1 = \angle BB_1B_2 = \angle BB_2B_1 = \angle CC_1C_2 = \angle CC_2C_1.$$
The lines AA_1, BB_1 and CC_1 bound a triangle, and the lines AA_2, BB_2 and CC_2 bound a second triangle. Prove that all six vertices of these two triangles lie on a single circle.
6. Find the value of the continued root :
$$\sqrt{4+27\sqrt{4+29\sqrt{4+31\sqrt{4+33\sqrt{\ldots.}}}}}\ .$$

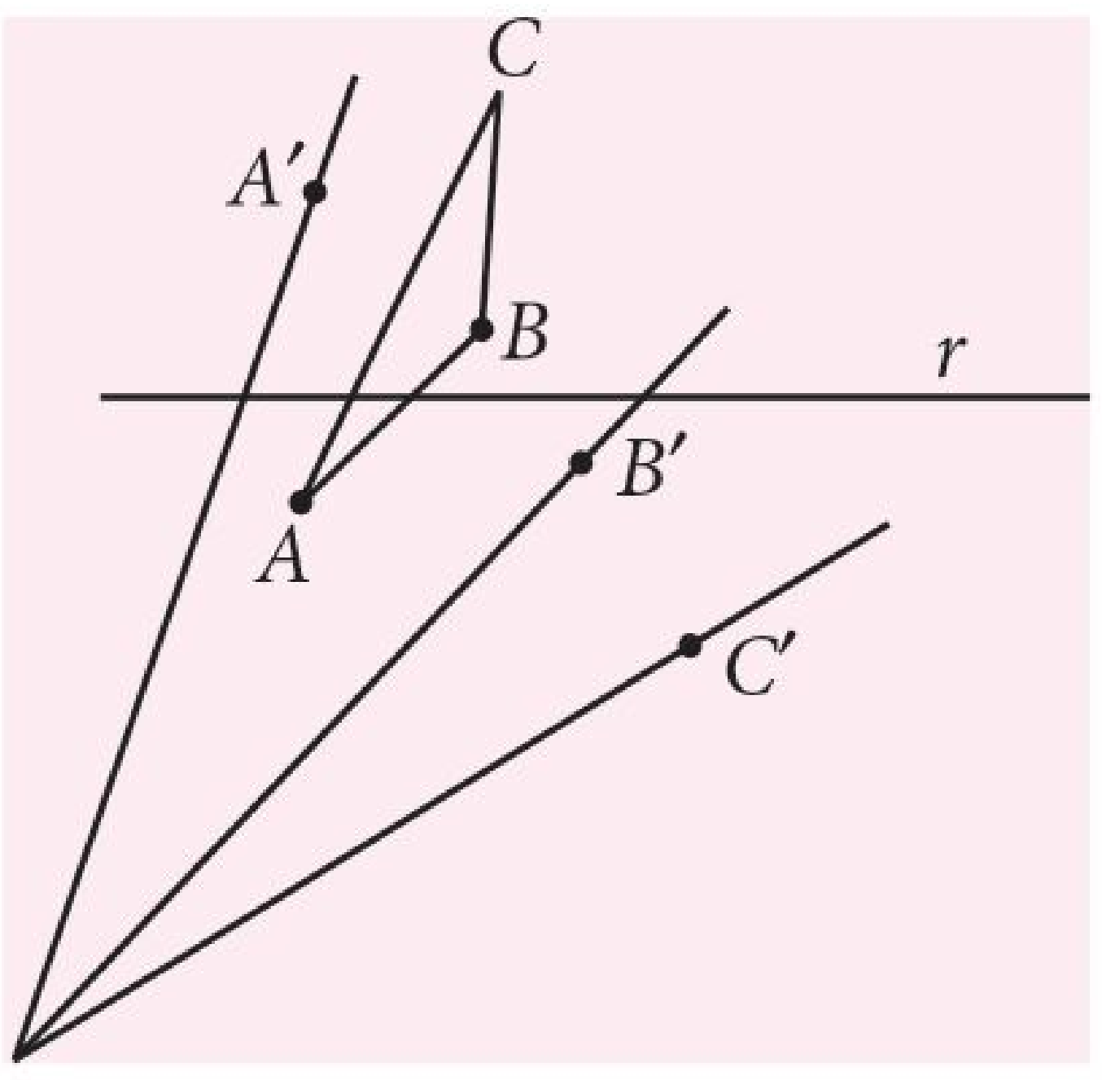
SOLUTIONS

1. This follows directly from Eisenstein's criterion, but we will take a more basic approach. By Demoivre's theorem, the n roots of $p_n(x)$ (in C) are $2^{\frac{1}{n}} \operatorname{cis}\left(\frac{2k\pi}{n}\right)$, $k = 0, 1, 2, \ldots n - 1$, so that
$$p_n(x) = \left(x - 2^{\frac{1}{n}} \operatorname{cis} 0\right)\left(x - 2^{\frac{1}{n}} \operatorname{cis}\left(\frac{2\pi}{n}\right)\right)\ldots. \left(x - 2^{\frac{1}{n}} \operatorname{cis}\left(\frac{2(n-1)\pi}{n}\right)\right)$$
Suppose $p_n(x) = f(x)\, g(x)$ for some $f(x), g(x) \in Q[x]$, with $\deg f(x) = m$ and $n > m > 0$. (Note : We only need to consider $n \geq 2$). Let the roots of $f(x)$, respectively $g(x)$, be $w_1, w_2, \ldots w_m$, respectively $w_{m+1}, w_{m+2}, \ldots w_n$, so $w_1, w_2, \ldots w_n$ is permutation of the roots of $p_n(x)$.

Let $w_i = 2^{1/n} \operatorname{cis} \theta_i$, and let $\theta = \sum_{k=1}^{m} \theta_i$.

Since, $(-1)^m \prod_{k=1}^{m} w_k$ is constant term of $f(x)$, so
$$(-1)^m \prod_{k=1}^{m} w_k = (-1)^m 2^{\frac{m}{n}} \prod_{k=1}^{m} \operatorname{cis} \theta_k = (-1)^m\, 2^{\frac{m}{n}} \operatorname{cis} \theta \in Q.$$
But $\operatorname{cis} \theta = \cos\theta + i \sin\theta \in Q$ implies that $\sin\theta = 0$ and further, that $\cos\theta = \pm 1$. Therefore, the constant term of $f(x)$ is $\pm 2^{\frac{m}{n}}$. It is easy to show that $2^{\frac{m}{n}} \notin Q$, a contradiction. Hence, $x^n - 2$ is irreducible in $Q[x]$.

2. Here, there is no need that ΔABC be contained in one of the half planes determined by r.



If the coordinates of A are (x_1, y_1), the coordinates of B are (x_2, y_2) and those of C are (x_3, y_3) in a cartesian coordinate system with r as x-axis then A' has coordinates $(x_1, -y_1)$, B' has coordinates $(x_2, -y_2)$ and C' has coordinates $(x_3, -y_3)$.

The equation of the line through A' parallel to BC is
$$(y_2 - y_3)x - (x_2 - x_3)y - x_1(y_2 - y_3) - y_1(x_2 - x_3) = 0$$
The equation of the line through B' parallel to CA is

$(y_3 - y_1)\,x - (x_3 - x_1)\,y - x_2\,(y_3 - y_1) - y_2\,(x_2 - x_1) = 0$

The equation of the line through C' parallel to AB is

$(y_1 - y_2)\,x - (x_1 - x_2)\,y - x_3\,(y_1 - y_2) - y_3\,(x_1 - x_2) = 0$

Since the three equations when added together vanish identically, the lines represented by them meet in a point. Its coordinates are found, by solving between any two, to be

$$\left(\frac{(x_1^2+x_2x_3)(y_2-y_3)+(x_2^2+x_3x_1)(y_3-y_1)+(x_3^2+x_1x_2)(y_1-y_2)}{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)},\ \frac{(y_1^2+y_2y_3)(x_2-x_3)+(y_2^2+y_3y_1)(x_3-x_1)+(y_3^2+y_1y_2)(x_1-x_2)}{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)}\right)$$

3. The squares can only be 0, 1, 4 or 7 mod 9.

Thus the sum of the digits of a perfect square cannot be 2, 3, 5, 6 or 8 mod 9, since the number itself would then be 2, 3, 5, 6 or 8 mod 9.

We shall show that the sum of the digits of a perfect square can take every value of the form 0, 1, 4 or 7 mod 9.

$$(10^m-1)^2 = 10^{2m} - 2\cdot 10^m + 1 = \underbrace{9\ 9\ldots9}_{m-1}\ 8\ \underbrace{0\ldots0}_{m-1}\ 1,\ m \ge 1.$$

The sum of the digits is $9m$, giving all the values greater than or equal to 9 congruent to 0 mod 9

$$(10^m-2)^2 = 10^{2m} - 4\cdot 10^m + 4 = \underbrace{9\ 9\ldots9}_{m-1}\ 6\ \underbrace{0\ldots0}_{m-1}\ 4,\ m \ge 1.$$

The sum of the digits is $9m + 1$, which gives all values greater than or equal to 10 congruent to 1 mod 9

$$(10^m-3)^2 = 10^{2m} - 6\cdot 10^m + 9 = \underbrace{9\ 9\ldots9}_{m-1}\ 4\ \underbrace{0\ldots0}_{m-1}\ 9,\ m \ge 1.$$

The sum of the digits is $9m + 4$, which takes every value greater than or equal to 13 which is congruent to 4 mod 9

$$(10^m-5)^2 = 10^{2m} - 10^{m+1} + 25 = \underbrace{9\ldots9}_{m-1}\ 0\ \underbrace{0\ldots0}_{m-1}\ 2\ 5.$$

4. We use mathematical induction.

Let $S_n(x) = \sum_{k=1}^{n} \frac{\sin(2k-1)x}{(2k-1)}$.

$S_1(x) = \sin x > 0$ for $x \in (0, \pi)$. Thus the proposed inequality is true for $n = 1$. Let $S_r(x) > 0$ for $r = 1, 2, \ldots, n - 1$. We will deduce that $S_n(x) > 0$ for $x \in (0, \pi)$. Suppose that $S_n(x_0) \le 0$ for some $x_0 \in (0, \pi)$, and that $S_n(x)$ attains its minimum at $x = x_0$. Hence $\frac{d}{dx}[S_n(x)]_{x=x_0} = 0$.

That is, $S_n'(x_0) = \sum_{k=1}^{n} \cos((2k-1)x_0) = 0$,

so that, $2\sin x_0 S_n'(x_0) = \sum_{k=1}^{n} 2\cos((2k-1)x_0)\sin x_0$

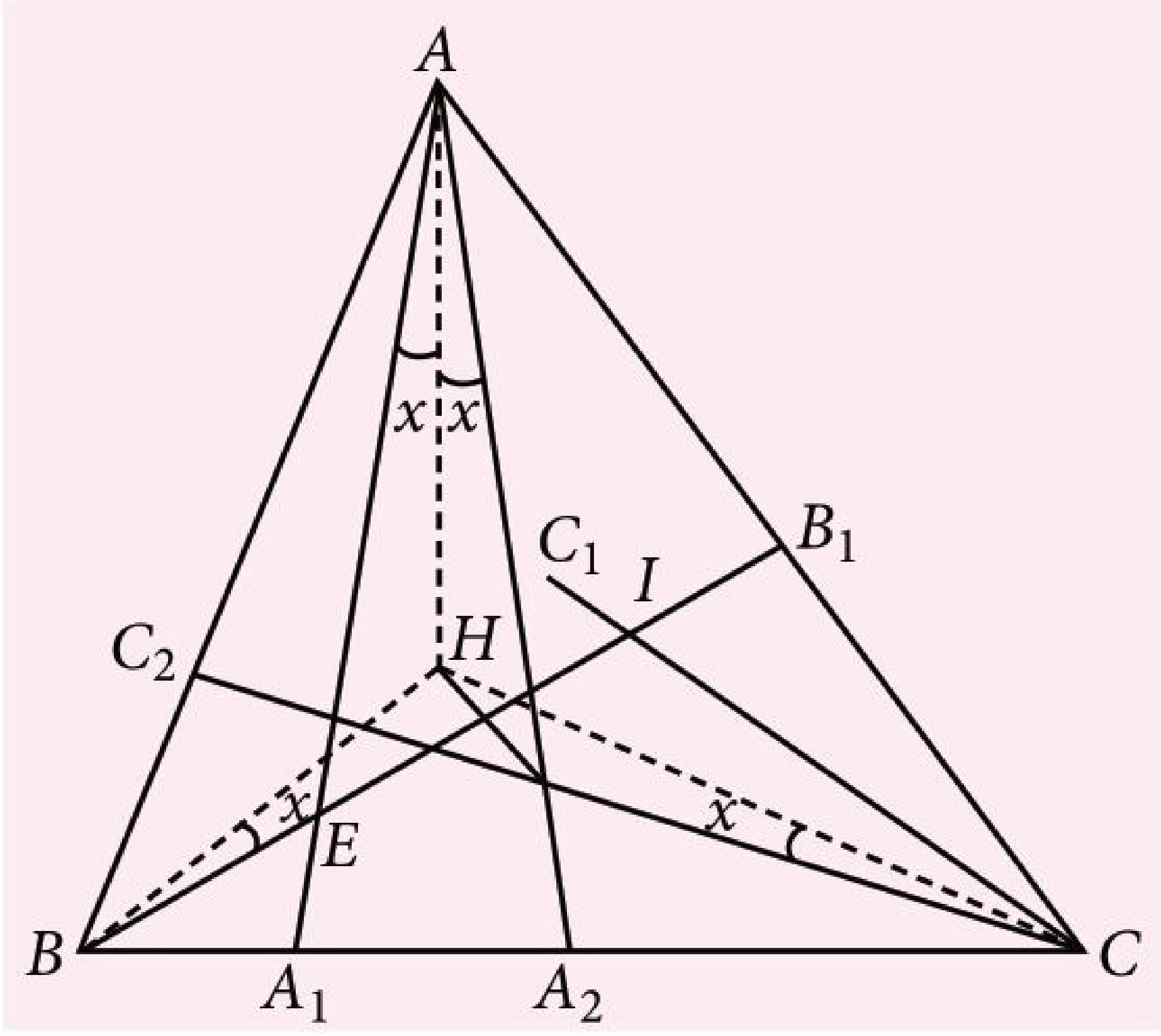
$= \sum_{k=1}^{n} [\sin(2kx_0) - \sin((2k-2)x_0)] = \sin 2nx_0$.

Thus $S_n'(x_0) = \frac{\sin 2nx_0}{2\sin x_0} = 0$ implying $\sin 2nx_0 = 0$.

Hence, $x_0 \in \left\{\frac{\pi}{2n}, \frac{2\pi}{2n}, \frac{3\pi}{2n}, \ldots, \frac{(2n-1)}{2n}\right\}$.

It is easily verified that at each of these values $S_n(x_0) > 0$, a contradiction. Hence $S_n(x) > 0$ for $x \in (0, \pi)$.

5.



Let AA_1, BB_1 meet at the point E; AA_1, CC_2 meet at the point F; and BB_1, CC_1 meet at the point I. Also $\angle A_1AA_2 = \angle B_1BB_2 = \angle C_1CC_2 = 2x$. (1)

The bisectors of the angles at A_1, B_1 and C_1 in triangles ΔA_1AA_2, ΔB_1BB_2 and ΔC_1CC_2 respectively are perpendicular to their respective bases. Hence they are the altitudes of ΔABC. Let H be the orthocentre of ΔABC.

Since $\angle A_1AH = \angle B_1BH = x$ and $\angle A_1AH = \angle C_1CH = x$ each one of the quadrilateral $AHEB$, $AHDC$ is inscribable in a circle.

These two circles have a common chord, the segment AH and since $\angle ABH = \angle ACH = 90° - \angle BAC$, then the circles have equal radii.

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)
6.	(a)	**7.**	(a, b)	**8.**	(b, c, d)	**9.**	(a, b)	**10.**	(a, b, c)
11.	(a, b, c, d)			**12.**	(c)	**13.**	(b, c)	**14.**	(c)
15.	(b)	**16.**	(a)	**17.**	(36)	**18.**	(3)	**19.**	(1)
20.	(1)								

Thus, since the inscribed angles $\angle EAH$, $\angle DAH$ are equal, the corresponding chords HE and HD are equal. Therefore $HE = HD$. Similarly, we prove that $HD = HI$, and so on for all six vertices of these two triangles of the problem.

Thus, all six vertices lie at the same distance from the point H, and the points are concyclic.

6. **I.** More generally, for any positive integer n, we claim that

$$\sqrt{4+n\sqrt{4+(n+2)\sqrt{4+(n+4)\sqrt{\ldots}}}} = n+2,$$

where the left side is defined as the limit of

$$F(n,m)=\sqrt{4+n\sqrt{4+(n+2)\sqrt{4+(n+4)\sqrt{\ldots\sqrt{4+m\sqrt{4}}}}}}$$

as $m \to \infty$ (where m is an integer and $(m-n)$ is even)

If $g(n, m) = F(n, m) - (n + 2)$, we have

$$F(n, m)^2 - (n+2)^2 = (4 + nF(n+2, m)) - (4 + n(n+4))$$

$$= n(F(n + 2, m) - (n + 4)),$$

so, $g(n,m)=\dfrac{n}{F(n,m)+n+2}\,g(n+2,m).$

Clearly $F(n, m) > 2$, so

$$\left|g(n,m)\right| < \frac{n}{n+4}\left|g(n+2,m)\right|.$$

By iterating this, we obtain

$$\left|g(n,m)\right| < \frac{n(n+2)}{m(m+2)}\left|g(m,m)\right| < \frac{n(n+2)}{m}.$$

Therefore $g(n, m) \to 0$ as $m \to \infty$

II. Let

$$S_n=\sqrt{4+(2n-1)\sqrt{4+(2n+1)\sqrt{4+(2n+3)\sqrt{\ldots}}}}$$

S_n satisfies the recurrence relation

$S_n=\sqrt{4+(2n-1)S_{n+1}}$ if and only if

$(S_n - 2)\,(S_n + 2) = (2n - 1)S_{n+1}$

By inspection, this admits $S_n = 2n + 1$ as a solution. We only have to prove that $S_1 = 3$ to make this induction complete. Let

$$T_n=\sqrt{4+\sqrt{4+3\sqrt{\ldots(2n-3)\sqrt{4+(2n-1)\sqrt{(2n+3)}}}}}$$

$$U_n=\sqrt{4+\sqrt{4+3\sqrt{\ldots(2n-3)\sqrt{4+(2n-1)(2n+3)}}}} = 3.$$

Clearly $T_n \le U_n$ and the latter is identically equal to 3. Therefore, using the fact that $B \ge A > 0$ implies that $\sqrt{(4+A)/(4+B)} \ge \sqrt{A/B}$

$$1 \ge \frac{T_n}{3} = \frac{T_n}{U_n} = \frac{\sqrt{4+\sqrt{\ldots+(2n-1)\sqrt{(2n+3)}}}}{\sqrt{4+\sqrt{\ldots+(2n-1)(2n+3)}}}$$

$$\ge \frac{\sqrt{\sqrt{\ldots+(2n-1)\sqrt{(2n+3)}}}}{\sqrt{\sqrt{\ldots+(2n-1)(2n+3)}}} \ge \ldots \ge \sqrt[2]{\frac{1}{2n+3}}$$

$$= \frac{1}{(2n+3)^{\left(\frac{1}{2}\right)}} \to 1$$

as $n \to \infty$ [for example, by rewriting as $\exp\{-\ln(2n+3)/2^{n+1}\}$ and using L'Hospital rule]. This proves that $S_1 = \lim\limits_{n\to\infty} T_n = 3$. The required expression is precisely S_{14} and hence its value is 29.

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SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

	1						4						9						1	
8		2				1		9				5		3				2		9
	3	4		8			6						4			5		3	6	
			7		8										7		3			
		5		3		6								8		9		7		
			9		2										2		5			
	8			9			5						7			3			2	
2		6				9		7				6		2				4		3
	9						2			5			3						9	
									3		6									
								3				1								
									1		4									
	9						6			8			9						6	
1		7				8		9				2		7				8		5
	6			1			7						1			5			2	
			1		7										9		6			
		3		5		7								3		7		6		
			9		2										1		8			
	3	6		4			2						3			1		4	7	
2		9				3		7				6		2				3		8
	5						1						8						9	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

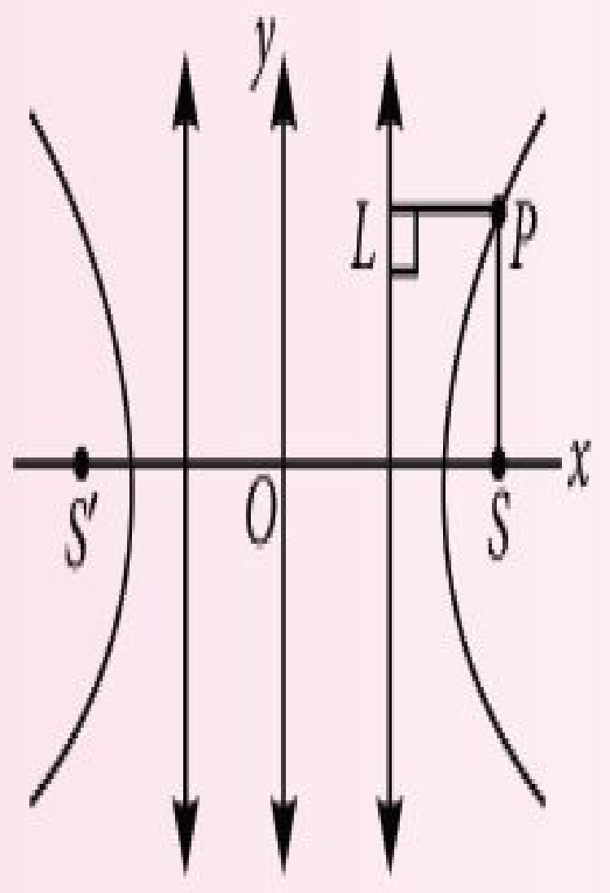
CONCEPT MAP

HYPERBOLA

Class XI

Definition

- Locus of a point which moves in such a way that the difference of its distance from two fixed points (foci) is always constant.



- General equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a hyperbola if $\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c\end{vmatrix}\neq 0$ and $h^2>ab$.

Standard Form

Standard form is given by $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

Centre : $O(0, 0)$

Foci : $S(ae, 0)$, $S_1(-ae, 0)$

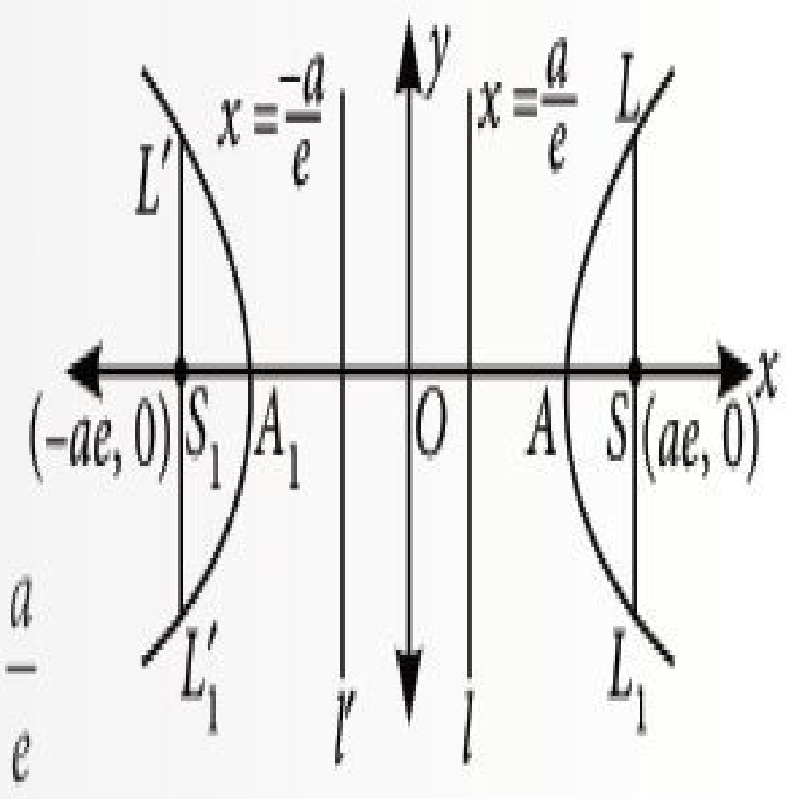
Vertices : $A(a, 0)$, $A_1(-a, 0)$

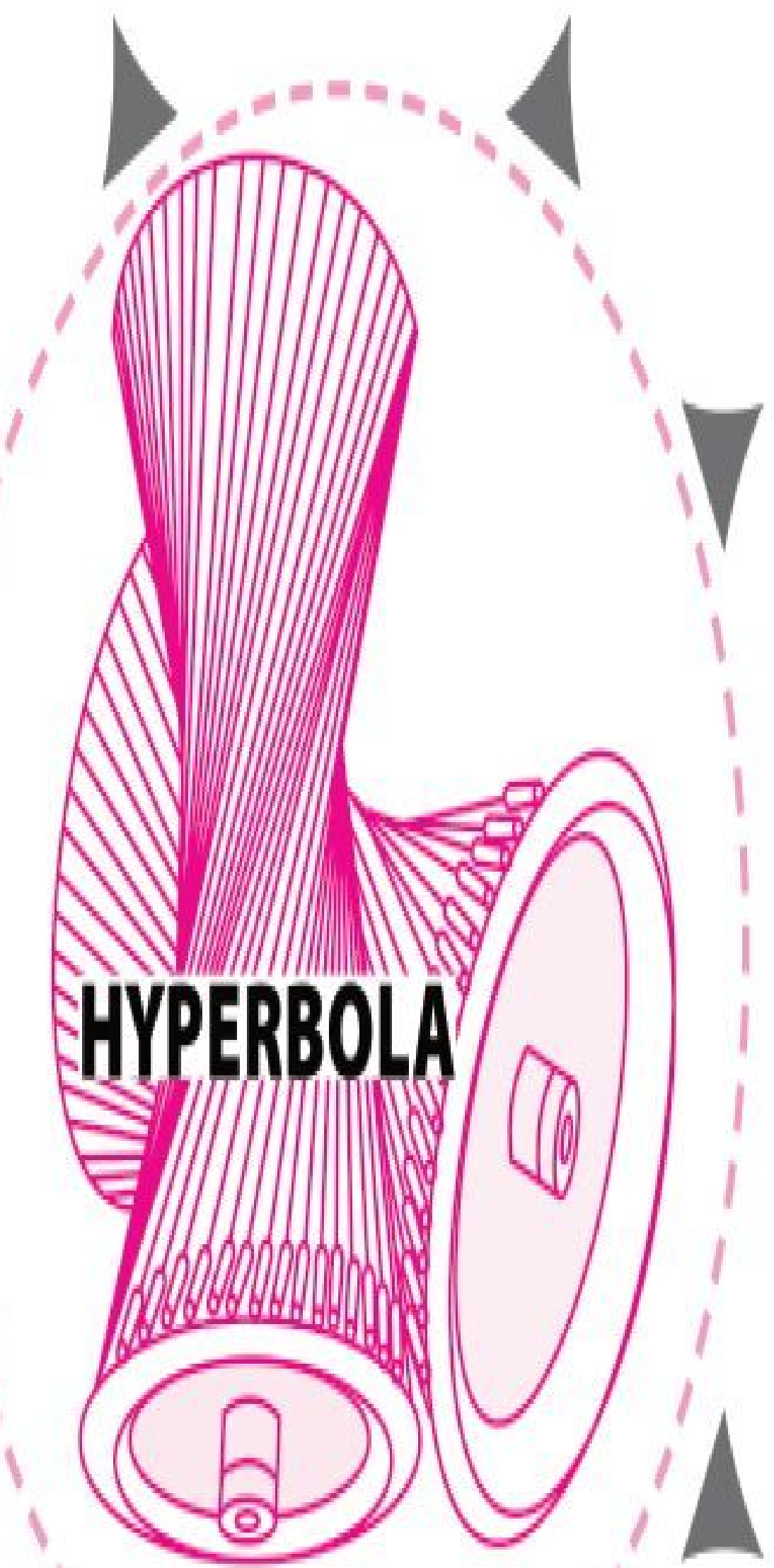
Directrices : $x=\dfrac{a}{e}$ and $x=-\dfrac{a}{e}$

Transverse axis : $AA_1 = 2a$

Conjugate axis = $2b$

Latus rectum : $LL_1 = L'L_1' = \dfrac{2b^2}{a} = 2a(e^2-1)$





Rectangular Hyperbola

Definition

A hyperbola is said to be rectangular hyperbola, if the asymptotes are at right angle *i.e.*, $\theta = 90°$. Equation of rectangular hyperbola is $x^2-y^2=a^2$ or $xy=c^2$.

Properties

If $A\left(ct_1, \dfrac{c}{t_1}\right)$ and $B\left(ct_2, \dfrac{c}{t_2}\right)$ are the points lying on the hyperbola $xy=c^2$ then

- Vertices of $xy=c^2$ are (c, c), $(-c, -c)$
- Foci : $(\pm c\sqrt{2}, \pm c\sqrt{2})$
- Equation of directrices is $x+y\pm c\sqrt{2}=0$
- Slope of chord at $A=-\dfrac{1}{t_1}$
- Chord subtend right angle at the vertices.
- Slope of tangent at $A=-\dfrac{1}{t_1^2}$
- Locus of the point of intersection of tangents at the end point of chords is $x+y=0$.

Congulated hyperbola

If $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ is one hyperbola, then its conjugate hyperbola is given by $\dfrac{y^2}{b^2}-\dfrac{x^2}{a^2}=1$

Important Result

$$S\equiv\frac{x^2}{a^2}-\frac{y^2}{b^2}-1,\ S_1\equiv\frac{x_1x}{a^2}-\frac{y_1y}{b^2}-1,$$

$$S_{11}\equiv\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1,\ S_2\equiv\frac{x_2x}{a^2}-\frac{y_2y}{b^2}-1,\ \ S_{22}\equiv\frac{x_2^2}{a^2}-\frac{y_2^2}{b^2}-1,\ S_{12}\equiv\frac{x_1x_2}{a^2}-\frac{y_1y_2}{b^2}-1,$$

- Location of $P(x_1, y_1)$: P is inside the hyperbola if $S_{11}>0$.
- P is outside hyperbola if $S_{11}<0$, P is on the hyperbola if $S_{11}=0$.
- Chord with midpoint $P(x_1, y_1)$: $S_1 = S_{11}$
- Chord joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$: $S_1+S_2=S_{12}$
- Tangent at $P(x_1, y_1)$: $S_1=0$
- Chord of contact of tangents from $P(x_1, y_1)$: $S_1=0$
- The tangents with slope m are $y=mx\pm\sqrt{a^2m^2-b^2}$
- Pair of tangents from $P(x_1, y_1)$: $S_1^2=S_{11}S$
- Normal at $P(x_1, y_1)$: $\dfrac{a^2x}{x_1}+\dfrac{b^2y}{y_1}=a^2+b^2=a^2e^2$

TANGENTS AND NORMALS

Class XII

CONCEPT MAP

Equation of Tangent and Normal

Equation of Tangent and Normal

For curve $y=f(x)$

- Equation of tangent at $A(x_1, y_1)$ is given by $y-y_1=m(x-x_1)$ or $y-y_1=\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}\cdot(x-x_1)$
- Equation of normal is given by

$$y-y_1=-\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x-x_1)$$

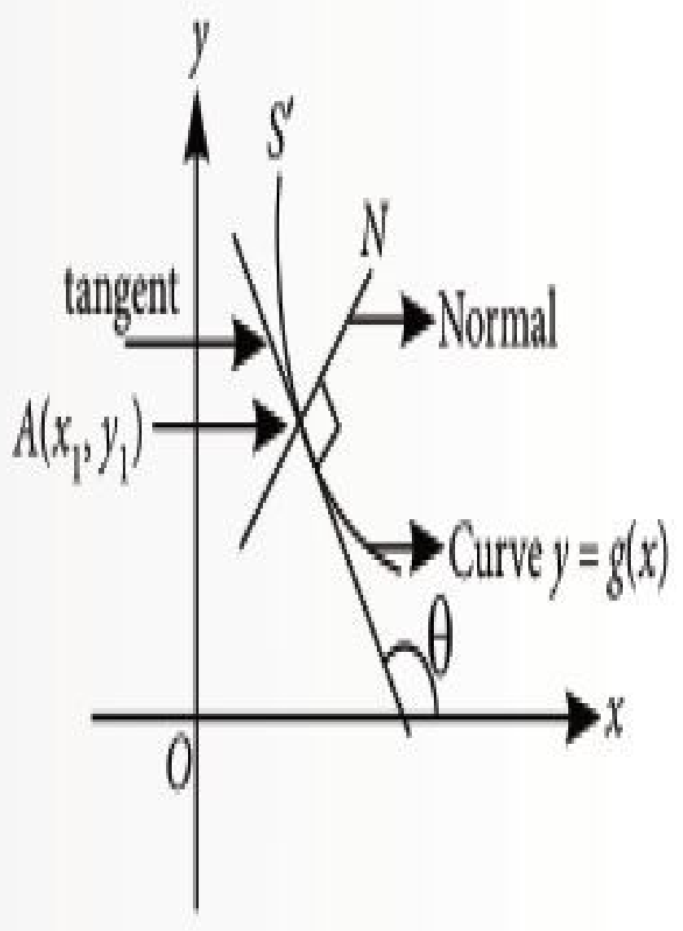
Note : (i) If $\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}=0$, then tangent at $A(x_1, y_1)$ is parallel to x-axis whose equation is $y=y_1$.

(ii) If $\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}=\infty$, then tangent at $A(x_1, y_1)$ is parallel to y-axis, whose equation is given by $x=x_1$.

Let $x=f(t)$, $y=g(t)$ be the equations of the curve in parametric form, then equation of tangent & normal are given by $y-g(t_1)=\dfrac{g'(t_1)}{f'(t_1)}(x-f(t_1))$ and $(y-g(t_1))\,g'(t_1)+(x-f(t_1))\,f'(t_1)=0$ respectively.

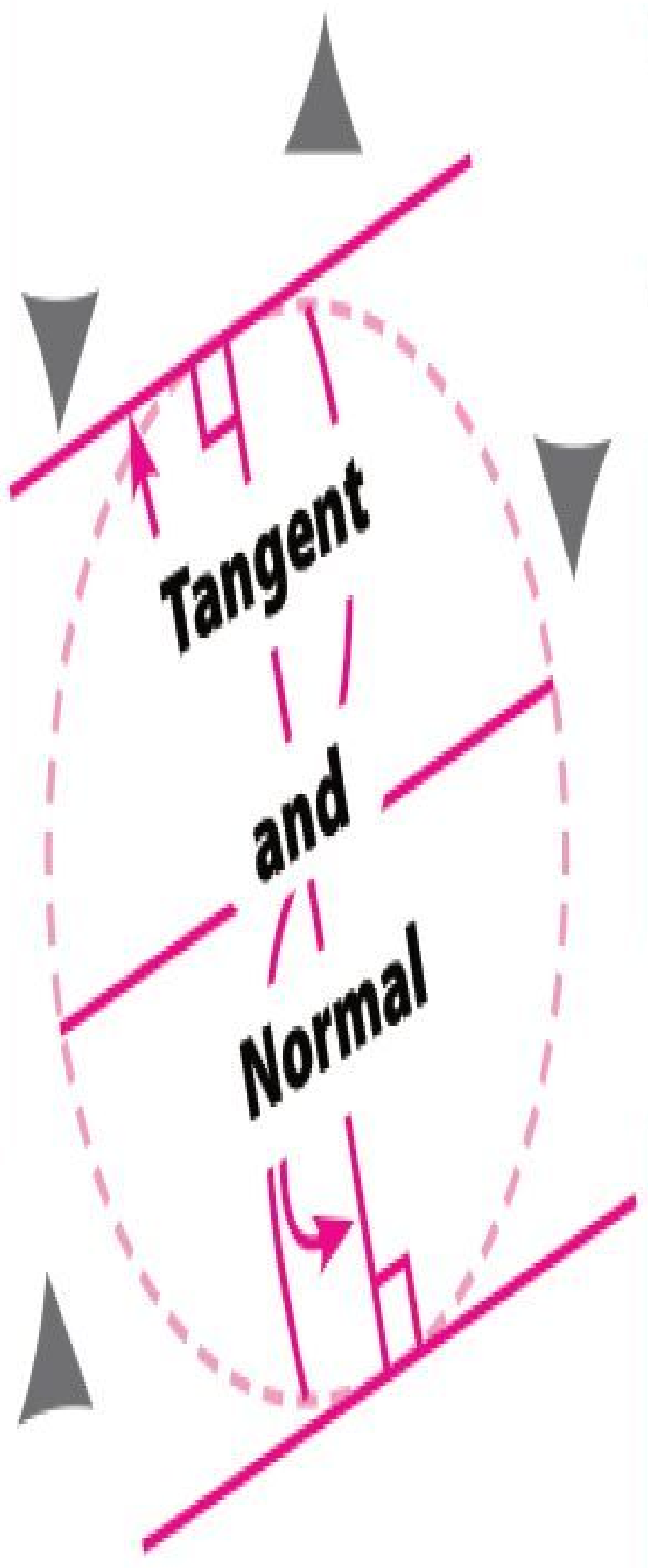
Slopes of Tangent and Normals

- **Slope of tangent :** Let $y=g(x)$ be a continuous curve & $A(x_1, y_1)$ be any point on the curve, then $\left(\dfrac{dy}{dx}\right)_{A(x_1,y_1)}$ is known as the slope of the tangent denoted by tan θ to the curve $y=g(x)$ at point A, where θ is the angle made by the tangent in the positive direction of x-axis.



- **Slope of normal :** Slope of normal at $A(x_1, y_1)$ is

$$=-\frac{1}{\text{Slope of tangent at }A(x_1,y_1)}=-\left(\frac{dx}{dy}\right)_{A(x_1,y_1)}$$



Angle of Intersection of Two Curves

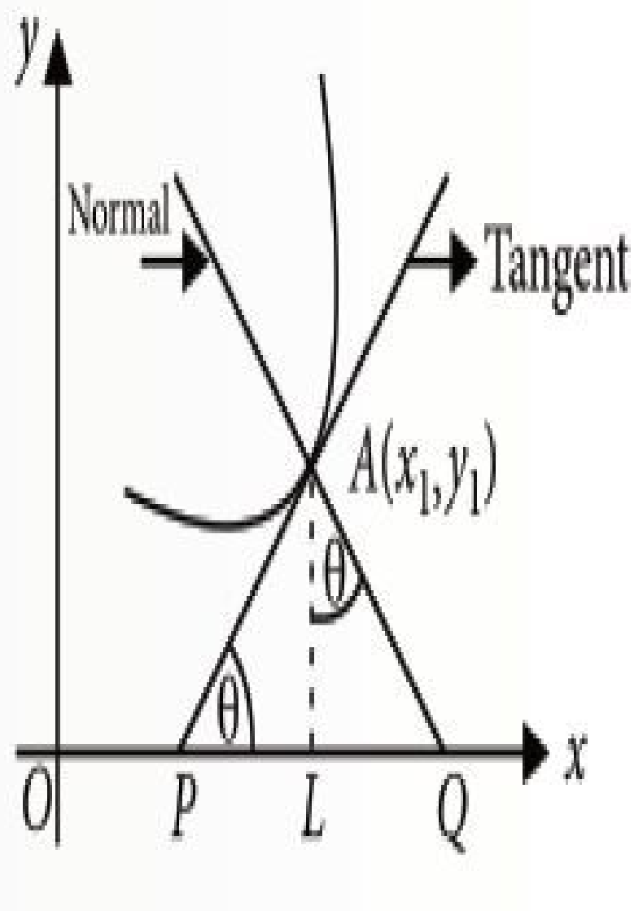
If θ be the angle of intersection of the curves having slope m_1 and m_2, then $\theta=\tan^{-1}\left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

- **Orthogonal Curves :** Two curves are said to be orthogonal if angle of their intersection is 90° *i.e.* $m_1m_2=-1$, where m_1, m_2 are their slopes respectively.
- **Touching Curves :** Two curves touch each other when $m_1=m_2$.

Some Important Points

If $m=\dfrac{dy_1}{dx_1}=\tan\theta$, then

- **Length of tangent** $=AP=|y_1\operatorname{cosec}\theta|=\left|y_1\sqrt{1+\cot^2\theta}\right|=\left|\dfrac{y_1\sqrt{1+\tan^2\theta}}{\tan\theta}\right|=\left|\dfrac{y_1\sqrt{1+\left(\frac{dy_1}{dx_1}\right)^2}}{\frac{dy_1}{dx_1}}\right|$
- **Length of normal** $=AQ=y_1\sec\theta=\left|y_1\cdot\sqrt{1+\tan^2\theta}\right|=\left|y_1\sqrt{1+\left(\dfrac{dy_1}{dx_1}\right)^2}\right|$
- **Length of subtangent** $=PL=|y_1\cot\theta|=\left|\dfrac{y_1}{\frac{dy_1}{dx_1}}\right|$ = Projection of tangent on the x-axis.
- **Length of subnormal** $=LQ=|y_1\tan\theta|=\left|y_1\dfrac{dy_1}{dx_1}\right|$
- Intercepts of tangent at (x_1, y_1) to the curve on x-axis $=x_1-y_1\left(\dfrac{dx_1}{dy_1}\right)$ and on y-axis $=y_1-x_1\left(\dfrac{dy_1}{dx_1}\right)$



JEE WORK CUTS

One or More Than One Option(s) Correct Type

1. The equation $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents a hyperbola with

(a) length of the transverse axis $= 2\sqrt{3}$

(b) length of the conjugate axis = 8

(c) centre at (1, –2) (d) eccentricity $= \sqrt{19}$

2. Which of the following functions is(are) periodic?

(a) Sgn (e^{-x}) (b) $\sin x + |\sin x|$

(c) min $(\sin x, |x|)$ (d) none of these

3. If $\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{A}[\log_e(f(x))/\log_e g(x)] + c$, then

(a) $A = \sin(a - b)$ (b) $f(x) = \sec(x - b)$

(c) $g(x) = \sec(x - a)$ (d) None of these.

4. If a circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches $x + y = 1$ and $x - y = 1$, then the centre of the circle is

(a) (4, 0) (b) (4, 2) (c) (6, 0) (d) (7, 9)

5. In a triangle ABC, if $a = \sqrt{6}, c = \sqrt{8}$ and $A = \frac{\pi}{4}$, its area is

(a) $2 - \sqrt{2}$ (b) $3 - \sqrt{2}$

(c) $2 + \sqrt{2}$ (d) $3 + \sqrt{2}$

6. If $21(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$, then x, y, z are in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

7. Let $f(x) = [9^x - 3^x + 1] \ \forall\, x \in (-\infty, 1)$, then range of $f(x)$ is ([·] denotes the greatest integer function)

(a) {0, 1, 2, 3, 4, 5, 6} (b) {0, 1, 2, 3, 4, 5, 6, 7}

(c) {1, 2, 3, 4, 5, 6} (d) {1, 2, 3, 4, 5, 6, 7}

8. If f is an even function defined on the interval (–5, 5), then the real values of x satisfying the condition $f(x) = f\left(\frac{x+1}{x+2}\right)$ are

(a) $\frac{-3 \pm \sqrt{5}}{2}, \frac{3 \pm \sqrt{5}}{2}$ (b) $\frac{-2 \pm \sqrt{3}}{2}, \frac{-3 \pm \sqrt{5}}{2}$

(c) $\frac{2 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{5}}{2}$ (d) None of these

9. The solution of the system of equations $x + y = \frac{\pi}{2}$ and $\sin x + \sin y = \sqrt{2}$ is (x, y), then

(a) $x = 2k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - 2k\pi$

(b) $x = 2k\pi - \frac{\pi}{2}, y = \frac{\pi}{2} + x$

(c) $x = 2k\pi + \frac{\pi}{3}, y = \frac{\pi}{3} - 2k\pi$

(d) $x = k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - k\pi \ (\forall\, k \in I)$

10. The solution set of the equation $\log_x 2 \log_{2x} 2 = \log_{4x} 2$ is

(a) $\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}$ (b) $\left\{\frac{1}{2}, 2\right\}$

(c) $\left\{\frac{1}{4}, 2^2\right\}$ (d) None of these

11. Let $A(-3, 2), B(2, -3)$ be the vertices of ΔABC. If the centroid of this triangle moves on the line $3x + 2y = 5$, then locus of the vertex C is the line

(a) $3x + 2y = 15$ (b) $3x + 2y = 20$

(c) $2x + 3y = 20$ (d) $3x - 2y = 15$

12. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$, then $a_1 + a_3 + a_5 + ... + a_{37}$ equals to

(a) $2^{19}(2^{20} - 21)$ (b) $2^{20}(2^{19} - 19)$
(c) $2^{19}(2^{20} + 21)$ (d) none of these

13. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$
(c) $6x = 4y = 3z$ (d) $x = y = z$

14. Let $\Delta(x)=\begin{vmatrix}(x-2) & (x-1)^2 & x^3\\ (x-1) & x^2 & (x+1)^3\\ x & (x+1)^2 & (x+2)^3\end{vmatrix}$, then the co-efficient of x in $\Delta(x)$ is

(a) -3 (b) -2 (c) -1 (d) 0

15. The probability that out of 5 person, all having birthday in March, at least two have the same birthday, is

(a) $\dfrac{{}^{31}C_5}{(31)^5}$ (b) $\dfrac{(31)^5 - {}^{31}C_5}{(31)^5}$

(c) $\dfrac{(31)^3 \times {}^{31}C_2}{(31)^5}$ (d) none of these

16. Let $f(x)=\sin^{-1}\left\{\dfrac{2g(x)}{1+\{g(x)\}^2}\right\}$, then

(a) $f(x)$ is decreasing, if $g(x)$ is increasing and $|g(x)| > 1$
(b) $f(x)$ is increasing, if $g(x)$ is increasing and $|g(x)| \le 1$
(c) $f(x)$ is increasing, if $g(x)$ is decreasing and $|g(x)| > 1$
(d) $f(x)$ is increasing, if $g(x)$ is decreasing and $|g(x)| \le 1$

17. Which of the following functions are continuous on $(0, \pi)$?

(a) $\tan x$ (b) $\int_0^x t\sin\left(\dfrac{1}{t}\right)dt$

(c) $\begin{cases} 1, & 0 < x \le \dfrac{3\pi}{4}\\ 3\sin\dfrac{2}{9}x, & \dfrac{3\pi}{4} < x < \pi \end{cases}$

(d) $\begin{cases} x\sin x, & 0 < x \le \pi/2\\ \dfrac{\pi}{2}\sin(\pi + x), & \dfrac{\pi}{2} < x < \pi \end{cases}$

18. Which of the following is true for ΔABC ?

(a) $\sin A \sin B \sin C \le \dfrac{3\sqrt{3}}{8}$

(b) $\sin^2A + \sin^2B + \sin^2C \le \dfrac{9}{4}$

(c) $\sin A \sin B \sin C$ is always positive

(d) $\sin^2A + \sin^2B \le 1 + \cos C$

19. If in ΔABC, $\begin{vmatrix}\sin A & \sin B & \sin C\\ \sin B & \sin C & \sin A\\ \sin C & \sin A & \sin B\end{vmatrix} = 0$, then

(a) ΔABC is equilateral (b) $a^3 + b^3 + c^3 = 3abc$
(c) $\Sigma\sin A = 0$ (d) $\Sigma\sin^3 A - 3\,\Pi\sin A = 0$

20. If (α, α^2) lies inside the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$, $5x - 6y - 1 = 0$, then

(a) $2\alpha + 3\alpha^2 - 1 > 0$ (b) $\alpha + 2\alpha^2 - 3 > 0$
(c) $\alpha + 2\alpha^2 - 3 < 0$ (d) $6\alpha^2 - 5\alpha + 1 > 0$

Paragraph for Q. No. 21 to 23

Consider the function $f(x)$, a fourth degree polynomial such that $\lim\limits_{x\to -1}\dfrac{f(x)}{(x+1)^3} = 1$ and $f'''(0) = 0$.

21. The maximum value of $f(x)$ is

(a) $\dfrac{13}{4}$ (b) $\dfrac{17}{4}$ (c) $\dfrac{23}{4}$ (d) $\dfrac{27}{4}$

22. The subnormal of the curve $y = f(x)$ where it cuts the y-axis is

(a) 1 (b) $\dfrac{3}{2}$ (c) 2 (d) $\dfrac{1}{2}$

23. The number of inflection points of the curve $y = f(x)$ is

(a) 0 (b) 1 (c) 2 (d) 3

Comprehension Type

Paragraph for Q. No. 24 to 26

In a G.P., the sum of the first and last term is 66, the product of the second and the last before one is 128 and the sum of the terms is 126.

24. If an increasing G.P. is considered, then the number of terms in G.P. is

(a) 9 (b) 8 (c) 12 (d) 6

25. If the decreasing G.P. is considered, then the sum of infinite terms is

(a) 64 (b) 128 (c) 256 (d) 729

26. In any case, the difference of the least and greatest terms is

(a) 78 (b) 135
(c) 126 (d) None of these

Matrix Match Type

27. Match the following:

Column-I		Column-II	
(A)	$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{r}{\left(\begin{array}{l}\text{product of first } (r+1)\\ \text{positive odd integers}\end{array}\right)}$ is	(P)	6
(B)	If $N=\sum\limits_{a<b}(a-b)^2$ where $a, b \in \{1, 3,$ $5, ..., 19\}$ then the sum of the digits of N is	(Q)	3
(C)	The sum of infinite series $\dfrac{3}{1^2}+\dfrac{5}{1^2+2^2}+\dfrac{7}{1^2+2^2+3^2}+\ldots$ is	(R)	4
(D)	The sum of infinite series $1+\dfrac{2}{3}+\dfrac{6}{3^2}+\dfrac{10}{3^3}+\ldots.$ is	(S)	1/2

28. Match the following :

Column-I		Column-II	
(A)	The value of α for which the vectors $2\hat{i}-\hat{j}+\hat{k},\ \hat{i}+2\hat{j}+\alpha\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$ are coplanar is	(P)	4
(B)	The area of a parallelogram having adjacent sides $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}+4\hat{k}$ is	(Q)	–3
(C)	If $\vec{r}\cdot\vec{a}=0, \vec{r}\cdot\vec{b}=0$ and $\vec{r}\cdot\vec{c}=0$ for some non-zero vector $\vec{r}$, then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is	(R)	$10\sqrt{3}$
(D)	The volume of parallelepiped whose sides are given $\overrightarrow{OA}=2\hat{i}-3\hat{j}$, $\overrightarrow{OB}=\hat{i}+\hat{j}-\hat{k}$ and $\overrightarrow{OC}=3\hat{i}-\hat{k}$ is	(S)	0

Numerical Answer Type

29. If $\alpha = 2\sin^{-1}(2/3)$ and $\beta = 2\tan^{-1}9$, then $80\text{cosec}^2\alpha + 81\,\text{cosec}^2\beta$ is equal to ______.

30. Let $f(x)=\begin{vmatrix}\cos x & -x & 1\\ 2\sin x & -x^2 & 2x\\ \tan x & -x & 1\end{vmatrix}$.

The value of $\lim\limits_{x\to 0}\dfrac{f'(x)}{x}$ is ______.

31. The chord of the curve $y = -a^2x^2 + 5ax - 4$ touches the curve $y=\dfrac{1}{1-x}$ at the point $x = 2$ and is bisected by that point. Then the value of a is ______.

32. The number of values of θ in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ such that $\theta \ne \dfrac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is ______..

SOLUTIONS

1. **(a,b,c) :** $16x^2 - 3y^2 - 32x - 12y - 44 = 0$

$\Rightarrow\ 16(x^2 - 2x) - 3(y^2 + 4y) - 44 = 0$

$\Rightarrow\ 16\{(x-1)^2 - 1\} - 3\{(y+2)^2 - 4\} - 44 = 0$

$\Rightarrow\ 16(x-1)^2 - 3(y+2)^2 = 48$

$$\Rightarrow\ \frac{(x-1)^2}{(\sqrt{3})^2}-\frac{(y+2)^2}{4^2}=1$$

Now, length of transverse axis $= 2\times\sqrt{3} = 2\sqrt{3}$

Length of conjugate axis = 2 × 4 = 8

Centre is given by $x - 1 = 0$ and $y + 2 = 0$, *i.e.*, $(1, -2)$

Also, $4^2 = (\sqrt{3})^2(e^2-1) \Rightarrow e^2-1=\dfrac{16}{3}$

$\therefore\ e^2=\dfrac{19}{3} \Rightarrow e=\sqrt{\dfrac{19}{3}}$

2. **(a,b,c) :** (a) $\text{Sgn}(x)=\begin{cases}1, \text{ if } & x>0\\ 0, \text{ if } & x=0\\ -1, \text{if } & x<0\end{cases}$

$\therefore\ \text{Sgn}(e^{-x}) = 1 \qquad (\because\ e^{-x} > 0)$

$\Rightarrow$ $\text{Sgn}(e^{-x})$ is a constant function. Hence, it is periodic.

(b) $\because$ Period of $\sin x$ is 2π and period of $|\sin x|$ is π

$\therefore$ Period of $\sin x + |\sin x|$ is L.C.M. $\{2\pi, \pi\} = 2\pi$

(c) Let $f(x) = \min\{\sin x, |x|\} = \sin x\ [\because\ \sin x < |x|]$ and $\sin x$ is periodic with period 2π.

3. **(a,b,c) :** Let $I=\displaystyle\int\frac{dx}{\cos(x-a)\cos(x-b)}$

$$\Rightarrow\ I=\frac{1}{\sin(a-b)}\int\frac{\sin(x-b-(x-a))\,dx}{\cos(x-a)\cos(x-b)}$$

$$=\frac{1}{\sin(a-b)}\int(\tan(x-b)-\tan(x-a))\,dx$$

$$=\frac{1}{\sin(a-b)}\left\{\log\frac{\sec(x-b)}{\sec(x-a)}\right\}+c$$

$$=\frac{1}{A}\{\log_e f(x)/\log_e fg(x)\}+c$$

4. (a,c) : If centre is = $(h, 0)$, then

$$\frac{|h-0-1|}{\sqrt{2}}=\sqrt{(h-3)^2+\frac{7}{2}}$$

$\therefore \quad h = 6, 4$

Thus centres are $(6, 0)$ and $(4, 0)$.

5. (a, c): $a^2 = b^2 + c^2 - 2bc\cos A = b^2 + 8 - 4b$

$a = \sqrt{6} \Rightarrow b^2 - 4b + 2 = 0 \quad \therefore \quad b = 2 \pm \sqrt{2}, \Delta = \frac{1}{2}bc\sin A$

$\therefore \quad \Delta = \frac{1}{2}(2 \pm \sqrt{2})\sqrt{8} \times \frac{1}{\sqrt{2}} = 2 \pm \sqrt{2}$

6. (b) : $\because 21 = 1^2 + 2^2 + 4^2$

Now, $\quad (1^2 + 2^2 + 4^2)(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$

$\Rightarrow \quad (1^2 + 2^2 + 4^2)(x^2 + y^2 + z^2) - (1\cdot x + 2\cdot y + 4\cdot z)^2 = 0$

$$\Rightarrow \quad \begin{vmatrix} 1 & 2 \\ x & y \end{vmatrix}^2 + \begin{vmatrix} 2 & 4 \\ y & z \end{vmatrix}^2 + \begin{vmatrix} 4 & 1 \\ z & x \end{vmatrix}^2 = 0$$

$\Rightarrow \quad (y - 2x)^2 + (2z - 4y)^2 + (4x - z)^2 = 0$

which is possible only when

$y - 2x = 0,\ 2z - 4y = 0$ and $4x - z = 0$

or $\quad \frac{y}{x} = 2, \frac{z}{y} = 2, \frac{z}{x} = 4 \Rightarrow \frac{y}{x} = \frac{z}{y} \Rightarrow y^2 = zx$

$\therefore \quad x, y, z$ are in GP.

7. (a) : $\because 9^x - 3^x + 1 = \left(3^x - \frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$$= \left(3^x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$$

$\therefore \quad$ For $x \in (-\infty, 1)$, $\frac{3}{4} \leq 9^x - 3^x + 1 < \frac{28}{4}$

Now, $\quad [9^x - 3^x + 1] = \{0, 1, 2, 3, 4, 5, 6\}$

$\therefore \quad R_f = \{0, 1, 2, 3, 4, 5, 6\}$

8. (a) : Given, $f(x) = f\left(\frac{x+1}{x+2}\right)$

$\therefore \quad f(-x) = f\left(\frac{-x+1}{-x+2}\right) = f(x)$

($\because f(x)$ is an even function)

$\Rightarrow \quad x = \frac{-x+1}{-x+2}$ and $-x = \frac{x+1}{x+2}$

$\Rightarrow \quad x^2 - 3x + 1 = 0$ and $x^2 + 3x + 1 = 0$

$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 = -1 + \frac{9}{4}$ and $\left(x + \frac{3}{2}\right)^2 = -1 + \frac{9}{4}$

$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2$ and $\left(x + \frac{3}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{5}}{2}$ and $x = \frac{-3 \pm \sqrt{5}}{2}$

9. (a) : We have, $\sin x + \sin y = \sqrt{2}$

$\Rightarrow \quad 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \sqrt{2}$

$\Rightarrow \quad \cos\left(\frac{x-y}{2}\right) = 1 \qquad \left(\because x + y = \frac{\pi}{2}\right)$

$\Rightarrow \quad x - y = 2\cdot 2\, k\pi = 4k\pi$

$\therefore \quad x = 2k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - 2k\pi, k \in I$

10. (a) : $\log_x 2 \log_{2x} 2 = \log_{4x} 2$

Here $x > 0$ and $x \neq 1, \frac{1}{2}, \frac{1}{4}\left\{\because \log_a x = \frac{1}{\log_x a}\right\}$

$\Rightarrow \quad \frac{1}{\log_2 x}\cdot\frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$

$\Rightarrow \quad \log_2 x\cdot(1 + \log_2 x) = 2 + \log_2 x$

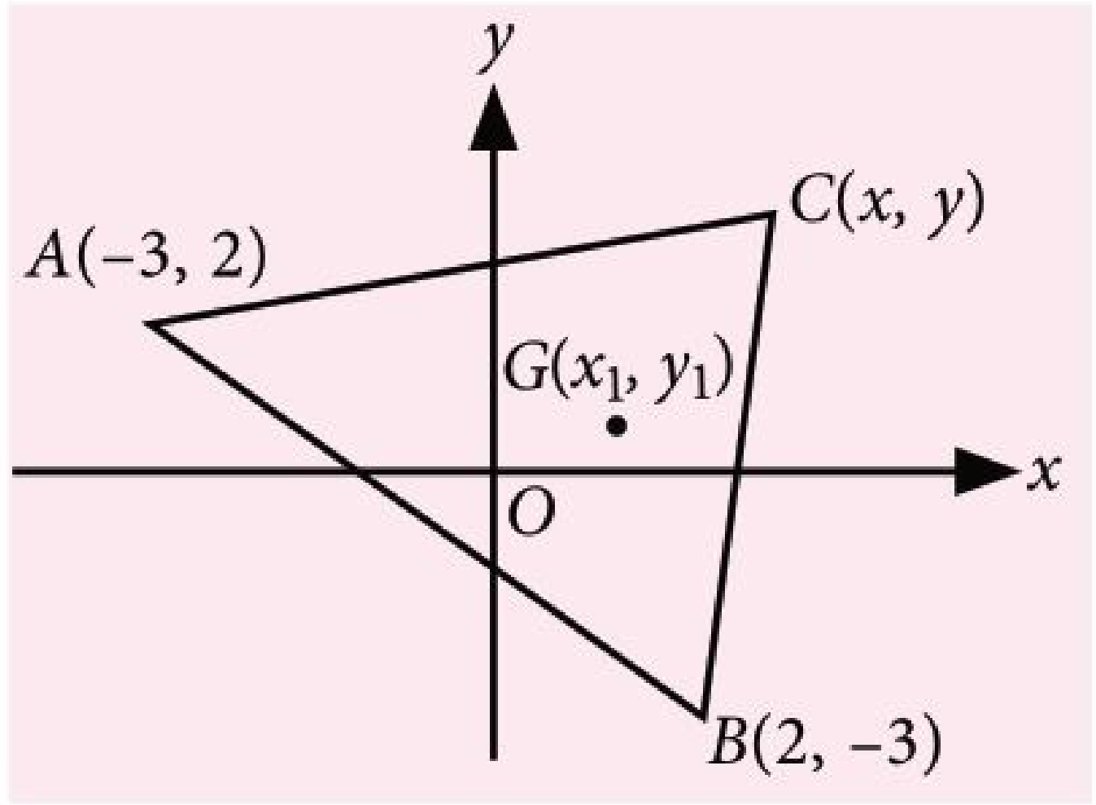
Put $\log_2 x = t$

$\Rightarrow \quad t^2 = 2 \quad \Rightarrow \quad t = \pm\sqrt{2}$

$\Rightarrow \quad \log_2 x = \pm\sqrt{2} \quad \Rightarrow \quad x = 2^{\pm\sqrt{2}}$

$\therefore \quad x = 2^{-\sqrt{2}}, 2^{\sqrt{2}}$

11. (b) : Let (x, y) be the co-ordinate of the vertex C and (x_1, y_1) be coordinate of centroid G.



$\therefore \quad x_1 = \frac{x-3+2}{3},$

$y_1 = \frac{y+2-3}{3}$

or $\quad (x_1, y_1) = \left(\frac{x-1}{3}, \frac{y-1}{3}\right)$

Now (x_1, y_1) lies on $3x + 2y = 5$

$\Rightarrow \quad 3x_1 + 2y_1 = 5$

$\Rightarrow \quad \frac{3(x-1)}{3} + \frac{2(y-1)}{3} = 5 \Rightarrow 3x + 2y - 3 - 2 = 15$

$\Rightarrow \quad 3x + 2y = 20$, which is required locus of vertex C.

12. (a) : $\because (1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \ldots + a_{37}x^{37} + a_{38}x^{38} + a_{39}x^{39} + a_{40}x^{40} \quad \ldots(i)$

Putting $x = 1$ and $x = -1$ in (i) and subtracting, we get

$4^{20} - 2^{20} = 2(a_1 + a_3 + a_5 + .. + a_{37} + a_{39})$

$\Rightarrow \quad a_1 + a_3 + a_5 + \ldots + a_{37} = 2^{19}(2^{20} - 1) - a_{39} \ \ldots(ii)$

Now, $(1 + x + 2x^2)^{20} = \frac{20!}{\alpha!\beta!\gamma!}(1)^\alpha(x)^\beta(2x^2)^\gamma$

For a_{39}, $\beta + 2\gamma = 39$ and $\alpha + \beta + \gamma = 20$

$\Rightarrow \quad \gamma = 19, \beta = 1, \alpha = 0$

$\therefore \quad a_{39} = \frac{20!}{0!1!19!}\cdot 2^{19} = 20\cdot 2^{19}$

$\therefore \quad$ From (ii), $a_1 + a_3 + a_5 + \ldots. + a_{37} = 2^{19}(2^{20} - 21)$

13. (d) : $\because$ As x, y, z are in A.P. $\therefore$ $2y = x + z$ ...(i)
$\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in A.P., then
$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$$\Rightarrow \quad 2\tan^{-1}y = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

Thus, $y^2 = xz$ ... (ii)
From (i) and (ii), we get $x = y = z$.

14. (b)

15. (b) : Number of days in month of March = 31
$\therefore$ $n(S) = (31)^5$
$n(A)$ = At least two have the same birthday
$= n(S)$ – Number of ways in which all candidates have the different birthday.
$= (31)^5 - {}^{31}C_5$

$$\therefore \quad P(A) = \frac{(31)^5 - {}^{31}C_5}{(31)^5}$$

16. (b,c) : $f(x) = \sin^{-1}(\sin 2\theta)$, where $g(x) = \tan\theta$
$= 2\theta$, $|g(x)| \le 1$
$= 2\tan^{-1}[g(x)], |g(x)| \le 1$

When, $|g(x)| > 1$, $f(x) = \sin^{-1}\left\{\frac{2/g(x)}{1+[1/g(x)]^2}\right\}$

$= \sin^{-1}(\sin 2\theta)$, where $\tan\theta = \dfrac{1}{g(x)}$
$= 2\theta = 2\cot^{-1}[g(x)]$, $|g(x)| > 1$

$$\therefore f'(x) = \begin{cases} \dfrac{2}{1+[g(x)]^2}\cdot g'(x), & |g(x)| \le 1 \\ \dfrac{-2}{1+[g(x)]^2}\cdot g'(x), & |g(x)| > 1 \end{cases}$$

17. (b) : The function $f(x) = \tan x$ is not defined at $x = \dfrac{\pi}{2}$, so $f(x)$ is not continuous on $(0, \pi)$.

For the function $f(x) = x\sin\dfrac{1}{x}$ is continuous on $(0, \pi)$ and the integral function of a continuous function is continuous, therefore $F(x) = \displaystyle\int_0^x t\sin\frac{1}{t}\,dt$ is continuous on $(0, \pi)$.

For the function $f(x) = \begin{cases} 1, & 0 < x \le \dfrac{3\pi}{4} \\ 3\sin\dfrac{2}{9}x, & \dfrac{3\pi}{4} < x < \pi \end{cases}$

$$\text{Now,} \quad \lim_{x\to\frac{3\pi}{4}^+} f(x) = \lim_{h\to 0} f\left(\frac{3\pi}{4}+h\right)$$

$$= \lim_{h\to 0} 3\sin\frac{2}{9}\left(\frac{3\pi}{4}+h\right)$$

$= 3\sin\pi/6 = \dfrac{3}{2}$ and $f(3\pi/4) = 1$

$\therefore$ $f(x)$ is discontinuous at $x = 3\pi/4$
$\Rightarrow$ $f(x)$ is discontinuous on $(0, \pi)$

For the function $F(x) = \begin{cases} x\sin x, & 0 < x \le \dfrac{\pi}{2} \\ \dfrac{\pi}{2}\sin(\pi + x), & \dfrac{\pi}{2} < x < \pi \end{cases}$

$$F(\pi/2) = \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2}$$

$$\text{Now,} \quad \lim_{x\to\frac{\pi}{2}^-} F(x) = \lim_{h\to 0} F\left(\frac{\pi}{2}-h\right)$$

$$= \lim_{h\to 0}\left(\frac{\pi}{2}-h\right)\sin\left(\frac{\pi}{2}-h\right) = \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2}$$

$$\lim_{x\to\frac{\pi}{2}^+} F(x) = \lim_{h\to 0} F\left(\frac{\pi}{2}+h\right) = \lim_{h\to 0}\frac{\pi}{2}\sin\left(\frac{3\pi}{2}+h\right)$$

$$= \frac{\pi}{2}\sin\frac{3\pi}{2} = -\frac{\pi}{2}$$

$\therefore$ $F(x)$ is not continuous at $x = \dfrac{\pi}{2}$.

18. (a,b,c,d) : $\sin^2 A + \sin^2 B + \sin^2 C$

$$= \frac{1}{2}(1-\cos 2A) + \frac{1}{2}(1-\cos 2B) + \sin^2 C$$

$$= 1 - \frac{1}{2}(\cos 2A + \cos 2B) + \sin^2 C$$

$$= 1 - \frac{1}{2}[2\cos(A+B)\cdot\cos(A-B)] + \sin^2 C$$

$= 1 + \cos C.\cos(A - B) + 1 - \cos^2 C$
$= 2 - \cos^2 C + \cos C\cdot\cos(A - B)$
$\therefore$ $\sin^2 A + \sin^2 B + \sin^2 C \le 2 - \cos^2 C + \cos C$ ...(i)
$= 2 - (\cos^2 C - \cos C)$

$$= 2 - \left(\cos^2 C - \cos C + \frac{1}{4}\right) + \frac{1}{4} = \frac{9}{4} - \left(\cos C - \frac{1}{2}\right)^2$$

$\therefore$ $\sin^2 A + \sin^2 B + \sin^2 C \le \dfrac{9}{4}$ ...(ii)

Now, for positive quantities A.M. ≥ G.M.

$$\therefore \quad \frac{\sin^2 A + \sin^2 B + \sin^2 C}{3} \ge \sqrt[3]{\sin^2 A \sin^2 B \sin^2 C}$$

$\therefore$ From eq.(ii),

$$\frac{9/4}{3} \ge \frac{\sin^2 A + \sin^2 B + \sin^2 C}{3} \ge (\sin A \sin B \sin C)^{2/3}$$

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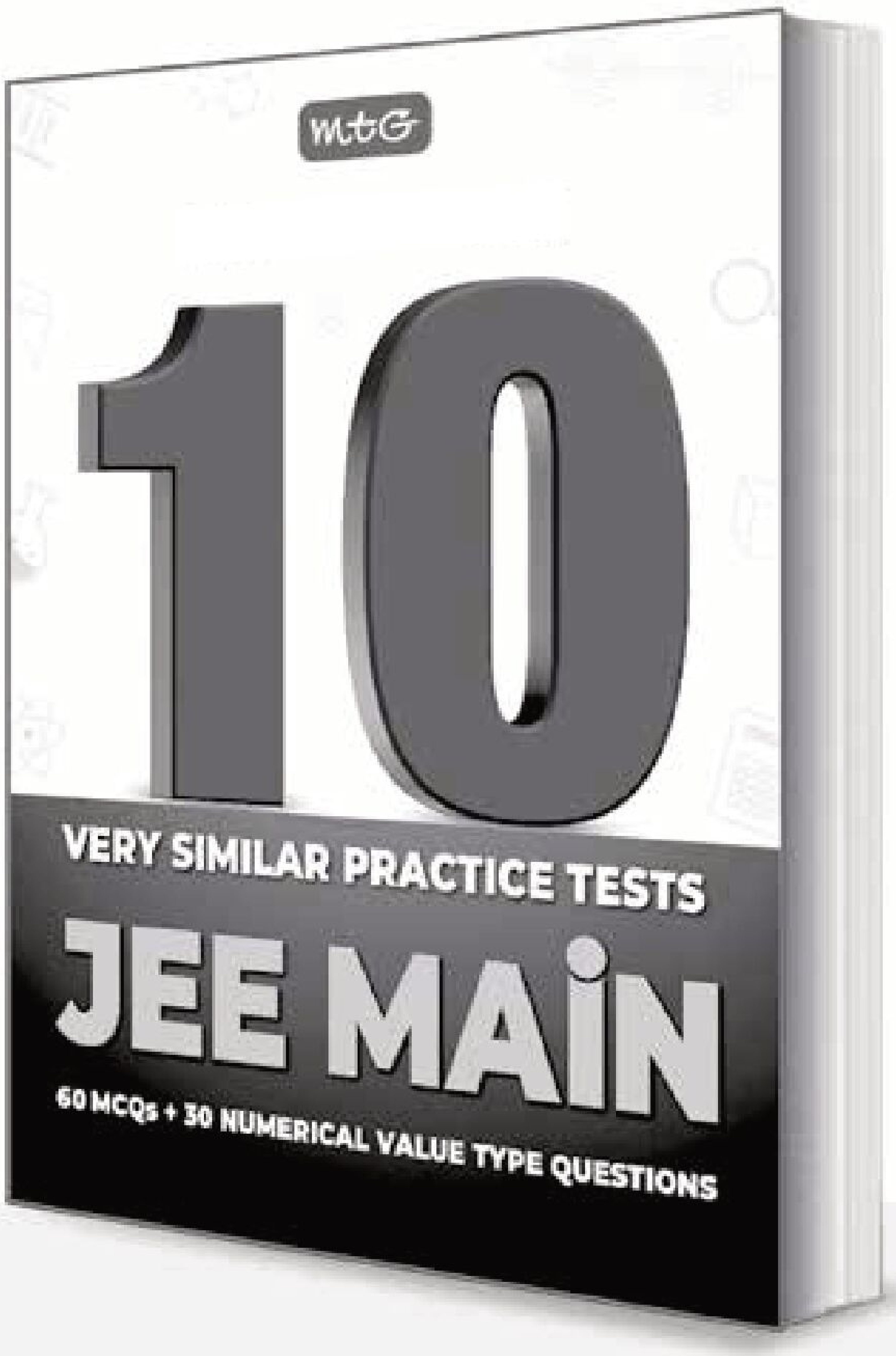
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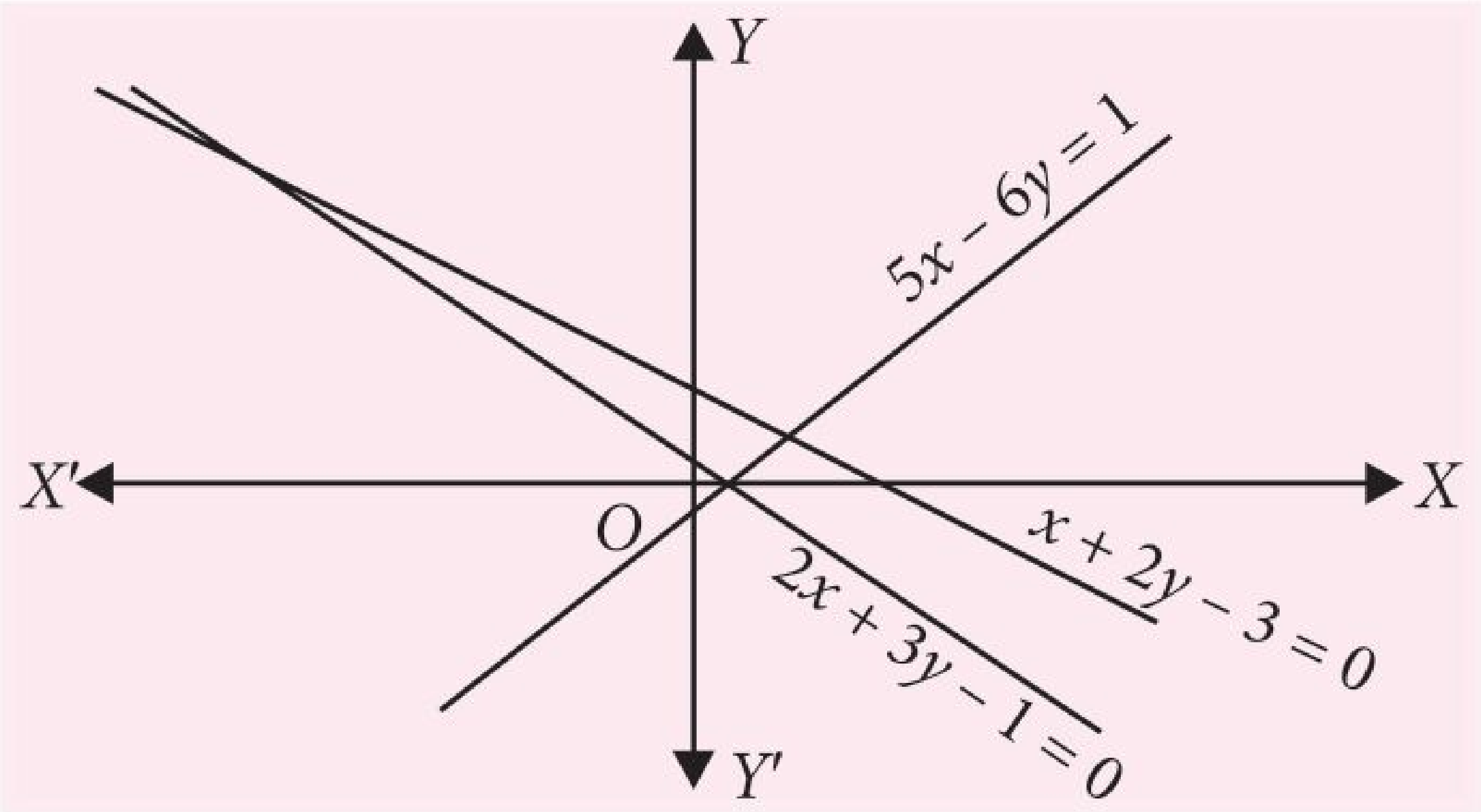
$$\Rightarrow \frac{3}{4} \geq (\sin A \sin B \sin C)^{2/3}$$

$$\text{or } \sin A \sin B \ \sin C \leq \left(\frac{3}{4}\right)^{3/2} = \frac{3\sqrt{3}}{8}$$

Also, in ΔABC, $\sin A > 0$, $\sin B > 0$, $\sin C > 0$ and from eq.(i), $\sin^2 A + \sin^2 B \leq 1 + \cos C$

19. (a, b,d)

20. (a,c,d):



The point (α, α^2) lie on the opposite sides with respect to $2x + 3y - 1 < 0$

Hence, $2\alpha + 3\alpha^2 - 1 > 0$

The point (α, α^2) lie on the same side with respect to $x + 2y - 3 < 0$. Hence, $\alpha + 2\alpha^2 - 3 < 0$

The point (α, α^2) lie on the same side with respect to $5x - 6y - 1 = 0$. Hence, $5\alpha - 6\alpha^2 - 1 < 0$

or $6\alpha^2 - 5\alpha + 1 > 0$

21. (d) : $\lim\limits_{x \to -1} \frac{f(x)}{(x+1)^3} = 1$ [Given]

$\Rightarrow \ f(-1) = f'(-1) = f''(-1) = 0, f'''(-1) = 6$

$\therefore \ f(x) = A(x + 1)^4 + (x + 1)^3$

$\Rightarrow \ f'''(x) = 24A(x + 1) + 6$

$f'''(0) = 0 \Rightarrow 24A + 6 = 0 \Rightarrow A = -\frac{1}{4}$

$\therefore \ f(x) = (x + 1)^3 - \frac{1}{4}(x + 1)^4$

$\Rightarrow \ f'(x) = 3(x + 1)^2 - (x + 1)^3$

$\Rightarrow \ f''(x) = 6(x + 1) - 3(x + 1)^2$

$f'(x) = 0 \Rightarrow (x + 1)^2(2 - x) = 0$

$\Rightarrow \ x = -1, x = 2$ and $f''(2) = -9$

So, $x = 2$ is a point of maxima.

Thus, the maximum value is $f(2) = 27 - \frac{81}{4} = \frac{27}{4}$

22. (b) : $f(0) = 1 - \frac{1}{4} = \frac{3}{4}, f'(0) = 2$

Subnormal at $x = 0$ is $f(0)\,f'(0) = \frac{3}{4} \times 2 = \frac{3}{2}$

23. (c): $f''(x) = 3(x + 1)(2 - x - 1) = 3(x + 1)(1 - x)$

$f''(x) = 0 \Rightarrow x = 1, -1$

24. (d) **25. (b)** **26. (d)**

27. (A) → (S); (B) → (P); (C) → (P); (D) → (Q)

A. We have,

$$t_r = \frac{r}{1\cdot3\cdot5\ldots\cdot(2r+1)} = \frac{1}{2}\left[\frac{2r+1-1}{1\cdot3\cdot5\ldots(2r+1)}\right]$$

$$\therefore\ t_r = \frac{1}{2}\left[\frac{1}{1\cdot3\cdot5\ldots(2r-1)} - \frac{1}{1\cdot3\cdot5\ldots(2r+1)}\right]$$

$$= -\frac{1}{2}[V_n - V_{n-1}] \qquad \text{(Using } V_n \text{ method)}$$

$$\therefore\ \sum_{r=1}^{n} t_r = -\frac{1}{2}[V_n - V_0] = \frac{1}{2}\left(1 - \frac{1}{1\cdot3\cdot5\ldots(2n+1)}\right)$$

$$\therefore\ \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{1\cdot3\cdot5\ldots(2n+1)}\right) = \frac{1}{2}$$

B. $\because \ N = \sum\limits_{a<b}(a-b)^2, \ a, b \in \{1, 3, 5, \ldots, 19\}$

$$\therefore\ N = (1-3)^2 + (1-5)^2 + \ldots + (1-19)^2$$
$$+ (3-5)^2 + \ldots + (3-19)^2 + \ldots\ldots + (17-19)^2$$

$$\Rightarrow\ N = 9(1^2 + 3^2 + 5^2 + \ldots + 19^2) - 2\sum_{a<b} ab$$

$$= 10(1^2 + 3^2 + 5^2 + \ldots + 19^2) - (1 + 3 + 5 + \ldots + 19)^2$$

[Using 2 (product of numbers taken two at a time) = square of the sum of numbers – sum of their squares]

$$= 10\left[\frac{10\times(400-1)}{3}\right] - (10^2)^2$$

$$\left[\text{Using } 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(4n^2-1)}{3}\right]$$

$$= \frac{100\times399}{3} - 10000 = 13300 - 10000 = 3300$$

$\therefore$ Sum of digits of $N = 3 + 3 + 0 + 0 = 6$

C. Let $S_r = \frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots + \frac{2r+1}{1^2+2^2+\ldots+r^2}$

$$\therefore\ t_r = \frac{2r+1}{1^2+2^2+\ldots+r^2} = \frac{6(2r+1)}{r(r+1)(2r+1)}$$

$$= \frac{6}{r(r+1)} = 6\left[\frac{1}{r} - \frac{1}{r+1}\right]$$

$$\therefore\ \sum_{r=1}^{n} t_r = 6\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) \qquad \therefore\ S_n = 6\left[1 - \frac{1}{n+1}\right]$$

$\therefore\ S_\infty = 6(1 - 0) = 6$

D. Let $S_r = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots \infty$ ... (i)

$$\therefore \quad \frac{1}{3}S_r = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots.\infty \qquad \ldots \text{(ii)}$$

Subtracting (ii) from (i), we get

$$\frac{2}{3}S_r = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots..\infty$$

$$= \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots..\infty$$

$$= \frac{4}{3}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots.\infty\right) = \frac{4}{3} \times \frac{1}{\left(1 - \frac{1}{3}\right)} = \frac{4}{3} \times \frac{3}{2} = 2$$

$$\therefore \quad S_r = \frac{2 \times 3}{2} = 3$$

28. (A) → (Q); (B) → (R); (C) → (S); (D) → (P)

A. Since, vectors are coplanar

$$\therefore \quad \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & \alpha \\ 3 & -4 & 5 \end{vmatrix} = 0 \Rightarrow \alpha = -3$$

B. Area of parallelogram $= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{matrix} \right\| = 10\sqrt{3}$

C. Since, $\vec{a}, \vec{b}, \vec{c}$ are coplanar. $\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

D. Volume of parallelepiped $= \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} = 4$

29. (1762) : Here, $\alpha = 2\sin^{-1}\left(\frac{2}{3}\right)$

$$= \sin^{-1}\left[2 \times \frac{2}{3}\sqrt{1 - \frac{4}{9}}\right] = \sin^{-1}\frac{4\sqrt{5}}{9}$$

$\Rightarrow$ 81 $\sin^2\alpha = 80 \Rightarrow 80\ \text{cosec}^2\alpha = 81$

Also, $\beta = 2\tan^{-1}9 = \sin^{-1}\frac{2 \times 9}{1 + 81} = \sin^{-1}\frac{9}{41}$

$\Rightarrow \quad \sin\beta = \frac{9}{41} \quad \Rightarrow 81\ \text{cosec}^2\beta = 1681$

$\therefore \quad 80\text{cosec}^2\alpha + 81\text{cosec}^2\beta = 1681 + 81 = 1762$

30. (2) : Differentiating $-f(x)$ w.r.t. x, we get

$$-f'(x) = \begin{vmatrix} -\sin x & x & 1 \\ 2\cos x & x^2 & 2x \\ \sec^2 x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x & 2x & 2x \\ \tan x & 1 & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 0 \\ 2\sin x & x^2 & 2 \\ \tan x & x & 0 \end{vmatrix}$$

$$\Rightarrow \quad -\frac{f'(x)}{x} = \begin{vmatrix} -\sin x & 1 & 1 \\ 2\cos x & x & 2x \\ \sec^2 x & 1 & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 1 & 0 \\ 2\sin x & x & 2 \\ \tan x & 1 & 0 \end{vmatrix}$$

[∵ The second determinant is zero]

$$\Rightarrow \quad -\lim_{x \to 0}\frac{f'(x)}{x} = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{vmatrix} = -2$$

31. (1) : $y = \frac{1}{1 - x}$

At $x = 2$, $y = -1$

Now, $\frac{dy}{dx} = \frac{1}{(1 - x)^2} \quad \therefore \quad \left.\frac{dy}{dx}\right|_{(2,-1)} = 1$

So, equation of tangent to the curve is

$y + 1 = 1(x - 2)$ or $y = x - 3$

Solving this line with $y = -a^2x^2 + 5ax - 4$, we get

$x - 3 = -a^2x^2 + 5ax - 4$

or $a^2x^2 + x(1 - 5a) + 1 = 0$ ...(i)

Above equation has two roots x_1 and x_2 which are abscissas of point of intersection P and Q.

Mid point of PQ lies on $x = 2$

$\therefore \quad \frac{x_1 + x_2}{2} = 2 \quad \Rightarrow \quad x_1 + x_2 = 4$

From equation (i), $x_1 + x_2 = -\left(\frac{1 - 5a}{a^2}\right) \Rightarrow 4 = \frac{5a - 1}{a^2}$

$\Rightarrow \quad 4a^2 - 5a + 1 = 0 \quad \Rightarrow \quad a = 1/4, 1$

But for $a = 1/4$, eq. (i) gives imaginary roots.

32. (3) : $\tan\theta = \cot 5\theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$

$\therefore \quad \theta = n\pi + \frac{\pi}{2} - 5\theta \Rightarrow \theta = (2n + 1)\frac{\pi}{12}$

$\theta = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{-\pi}{12}, \frac{-\pi}{4}, \frac{-5\pi}{12}$ ...(i)

$\cos 4\theta = \sin 2\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\therefore \quad 4\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \quad \theta = (4n + 1)\frac{\pi}{12}, (4n - 1)\frac{\pi}{4}$ ...(ii)

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{-\pi}{4}$

The common values of (i) and (ii) are in (ii). The number of values of θ is 3.

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YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then prove that $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2 \le 9.$ ***(Anuradha, Delhi)***

Ans. Let $\lambda = |\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$.

$$\Rightarrow \lambda = (|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b})+(|\vec{b}|^2+|\vec{c}|^2-2\vec{b}\cdot\vec{c}) + (|\vec{c}|^2+|\vec{a}|^2-2\vec{c}\cdot\vec{a})$$

$$\Rightarrow \lambda = \{2-2(\vec{a}\cdot\vec{b})\}+\{2-2(\vec{b}\cdot\vec{c})\}+\{2-2(\vec{c}\cdot\vec{a})\} \quad [\because |\vec{a}|=|\vec{b}|=|\vec{c}|=1]$$

$$\Rightarrow \lambda = 6-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$\Rightarrow 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 6-\lambda \quad \text{...(i)}$$

For any vector $\vec{r}$, we have $|\vec{r}|^2 \ge 0$.

$$\therefore \quad |\vec{a}+\vec{b}+\vec{c}|^2 \ge 0$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) \ge 0$$

$$\Rightarrow 3+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) \ge 0 \ [\because |\vec{a}|^2=|\vec{b}|^2=|\vec{c}|^2=1]$$

$$\Rightarrow 3+6-\lambda \ge 0 \quad \text{[From (i)]}$$

$$\Rightarrow 9-\lambda \ge 0 \ \Rightarrow \ \lambda \le 9$$

$$\Rightarrow |\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2 \le 9$$

2. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of points A, B, C, D respectively and $\vec{b}-\vec{a} = 2(\vec{d}-\vec{c})$, show that the point of intersection of the straight lines AD and BC divide these lines in the ratio 2:1. ***(Rajnish, Rajathan)***

Ans. Let P and Q be points which divide AD and BC respectively in the ratio 2 : 1.

Let the position vector of P and Q be $\vec{\alpha}$ and $\vec{\beta}$,respectively, then

$$\vec{\alpha} = \frac{2\vec{d}+\vec{a}}{2+1} = \frac{2\vec{d}+\vec{a}}{3} \quad \text{...(i)}$$

and $$\vec{\beta} = \frac{2\vec{c}+\vec{b}}{2+1} = \frac{2\vec{c}+\vec{b}}{3} \quad \text{...(ii)}$$

Given, $\vec{b}-\vec{a} = 2(\vec{d}-\vec{c}) \Rightarrow \vec{b} = \vec{a}+2\vec{d}-2\vec{c}$ ...(iii)

Putting this value of $\vec{b}$ in (ii), we get

$$\vec{\beta} = \frac{2\vec{c}+\vec{a}+2\vec{d}-2\vec{c}}{3} = \frac{2\vec{d}+\vec{a}}{3} = \vec{\alpha} \quad \text{[From (i)]}$$

Hence P and Q are the same points and point of intersection of AD and BC divides these lines in the ratio 2 : 1.

3. Find the value of $2\cot(\cot^{-1}(3)+\cot^{-1}(7)+\cot^{-1}(13)+\cot^{-1}(21))$. ***(Amitoj, U.P)***

Ans. Consider $\sum \cot^{-1}(n^2+n+1)$, where $n = 1, 2, 3, 4$

$$= \sum \tan^{-1}\frac{1}{1+n(n+1)} = \sum \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$$

$= \tan^{-1}(n+1) - \tan^{-1}(n) = T_n$, $n = 1, 2, 3, 4$

$\therefore \quad S = T_1 + T_2 + T_3 + T_4$

$$= (\tan^{-1}(2)-\tan^{-1}(1)) + (\tan^{-1}(3)-\tan^{-1}(2)) + (\tan^{-1}(4)-\tan^{-1}(3)) + (\tan^{-1}(5)-\tan^{-1}(4))$$

$$= \tan^{-1}(5)-\tan^{-1}(1) = \tan^{-1}\frac{5-1}{1+5} = \tan^{-1}\frac{2}{3}$$

$$\therefore \ 2\cot\left(\cot^{-1}\frac{3}{2}\right) = 2\cdot\frac{3}{2} = 3.$$

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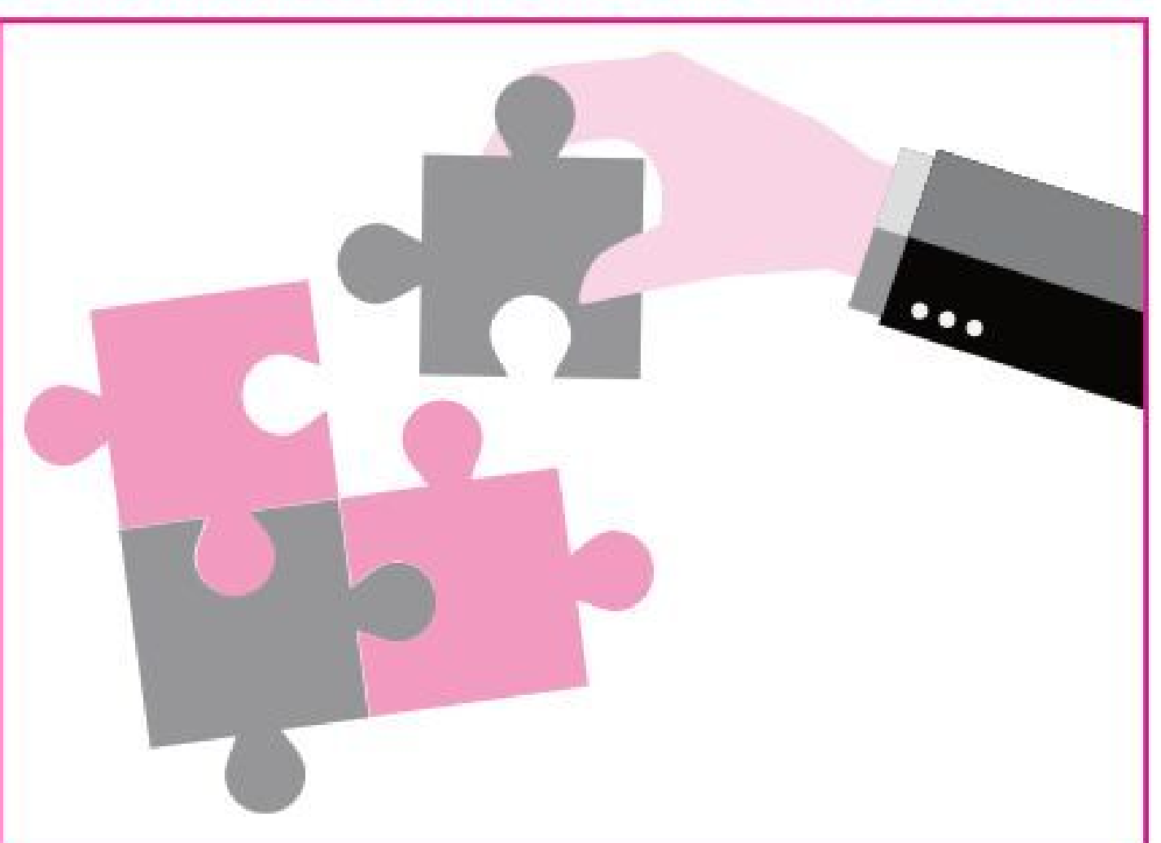
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

7+	15+		9+	2÷	
	1−				6+
3÷		48×		1−	
	5+		5		3−
2−		25×	12×		
				1−	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

CBSE warm-up!

CLASS-XI

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Series-2

Trigonometric Functions | Complex Numbers and Quadratic Equations

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-*I *and* II.
2. *Section-I comprises of* 16 *very short answer type questions.*
3. *Section-II contains* 2 *case study-based questions.*

Part - B :

1. *It consists of three Sections-*III, IV *and* V.
2. *Section-*III *comprises of* 10 *questions of* 2 *marks each.*
3. *Section-*IV *comprises of* 7 *questions of* 3 *marks each.*
4. *Section-*V *comprises of* 3 *questions of* 5 *marks each.*
5. *Internal choice is provided in* 3 *questions of Section-*III, 2 *questions of Section-*IV *and* 3 *questions of Section-*V. *You have to attempt only one of the alternatives in all such questions.*

PART - A

SECTION - I

1. If $\tan\theta = \frac{-4}{3}$ where $\frac{\pi}{2} < \theta < \pi$, then find the value of $\sin\theta$.

2. If $\cos x = \frac{4}{5}$, where $x \in [0, \pi/2]$, then find the value of $\cos\left(\frac{x}{2}\right)$.

OR

Find in radians, the angle between the hour hand and the minute hand of a clock at half past three.

3. Evaluate : $(-5i)\left(\frac{1}{8}i\right)$.

4. If $(x + iy)(p + iq) = (x^2 + y^2)i$, then find the values of p and q.

OR

Find the value of $(-\sqrt{-1})^{4n+3}, n \in N$.

5. Evaluate $\cos 1° \cos 2° \cos 3° \ldots \cos 179°$.

6. Find the value of $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$.

OR

If $\tan\alpha = 1/7$, then find the value of $\cos 2\alpha$.

7. What is the reciprocal of $3+\sqrt{7}i$?

8. Find the value of $\sec\frac{2\pi}{3} + \operatorname{cosec}\frac{5\pi}{6}$.

OR

Find the value of $\sin 50° - \sin 70° + \sin 10°$.

9. Express $\sqrt{-3}\sqrt{-6}$ in the form of $x + iy$.

10. Express i^{-35} in the form of $a + ib$.

11. Evaluate $\sqrt{3}$ cosec 20° – sec 20°.

12. Find the value of $2\sin 15° \cos 75°$.

OR

Convert 240° into radian.

13. Convert $40° 20'$ into radian measure.

14. If $\sin\theta + \operatorname{cosec}\theta = 2$, then find the value of $\sin^2\theta + \operatorname{cosec}^2\theta$.

15. Evaluate $i^2 + i^4 + i^6 + ...$ upto $(2n + 1)$ terms.

16. Find the smallest positive integer n, for which $(1 + i)^{2n} = (1 - i)^{2n}$.

SECTION - II

Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each sub part carries 1 mark.

17. A teacher mentioned in his class that there is some $\theta \in R$ such that $\tan\theta = \frac{-5}{12}$, where θ lies in second quadrant. Then, he asks his students about some other values.

(i) What will be the value of $\sin\frac{\theta}{2}$?

(a) $\frac{5}{13}$ (b) $\frac{12}{13}$ (c) $-\frac{5}{\sqrt{26}}$ (d) $\frac{5}{\sqrt{26}}$

(ii) Value of $\cos\frac{\theta}{2}$ is

(a) $\frac{5}{\sqrt{26}}$ (b) $\frac{3}{\sqrt{26}}$ (c) $\frac{1}{\sqrt{26}}$ (d) $\frac{13}{\sqrt{26}}$

(iii) Find the value of $\sin 2\theta$.

(a) $\frac{25}{26}$ (b) $\frac{-120}{169}$ (c) $\frac{120}{169}$ (d) $\frac{-25}{26}$

(iv) What is the value of $\tan 2\theta$?

(a) $\frac{-120}{119}$ (b) $\frac{25}{144}$ (c) $\frac{-25}{144}$ (d) $\frac{120}{119}$

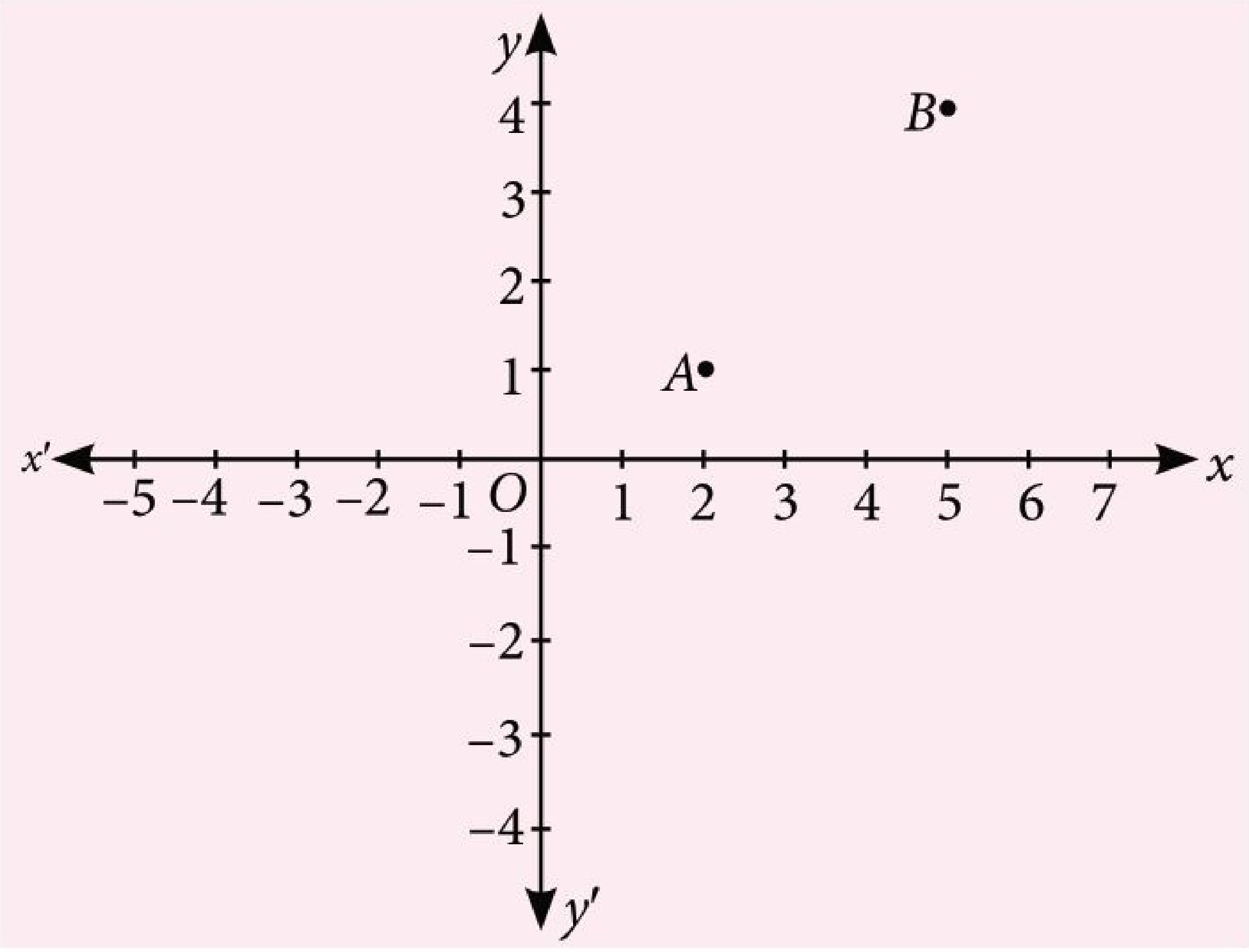
(v) Value of $\cos 3\theta$ will be

(a) $\frac{1728}{2197}$ (b) $-\frac{828}{2197}$

(c) $\frac{2128}{2197}$ (d) $\frac{-1728}{2197}$

18. Azim and Prakash, both studying in class XI, were discussing complex numbers. They planned to mix the coordinate geometry and complex numbers. For this, they formed a board game having an argand plane on it (as shown in the figure given below).

Later they took two dice of different colours. One dice of green colour and another one of black colour. Numbers on green dice represented real part whereas numbers on black die represented imaginary part. Both of them threw the dice and recorded the numbers as A and B. Based on the above information, answer the following questions.



(i) Choose the correct options which represents the complex numbers by the points A and B.

(a) $1 + 2i, 4 + 5i$ (b) $2 - i, 5 - 4i$

(c) $2 - i, 5 + 4i$ (d) $2 + i, 5 + 4i$

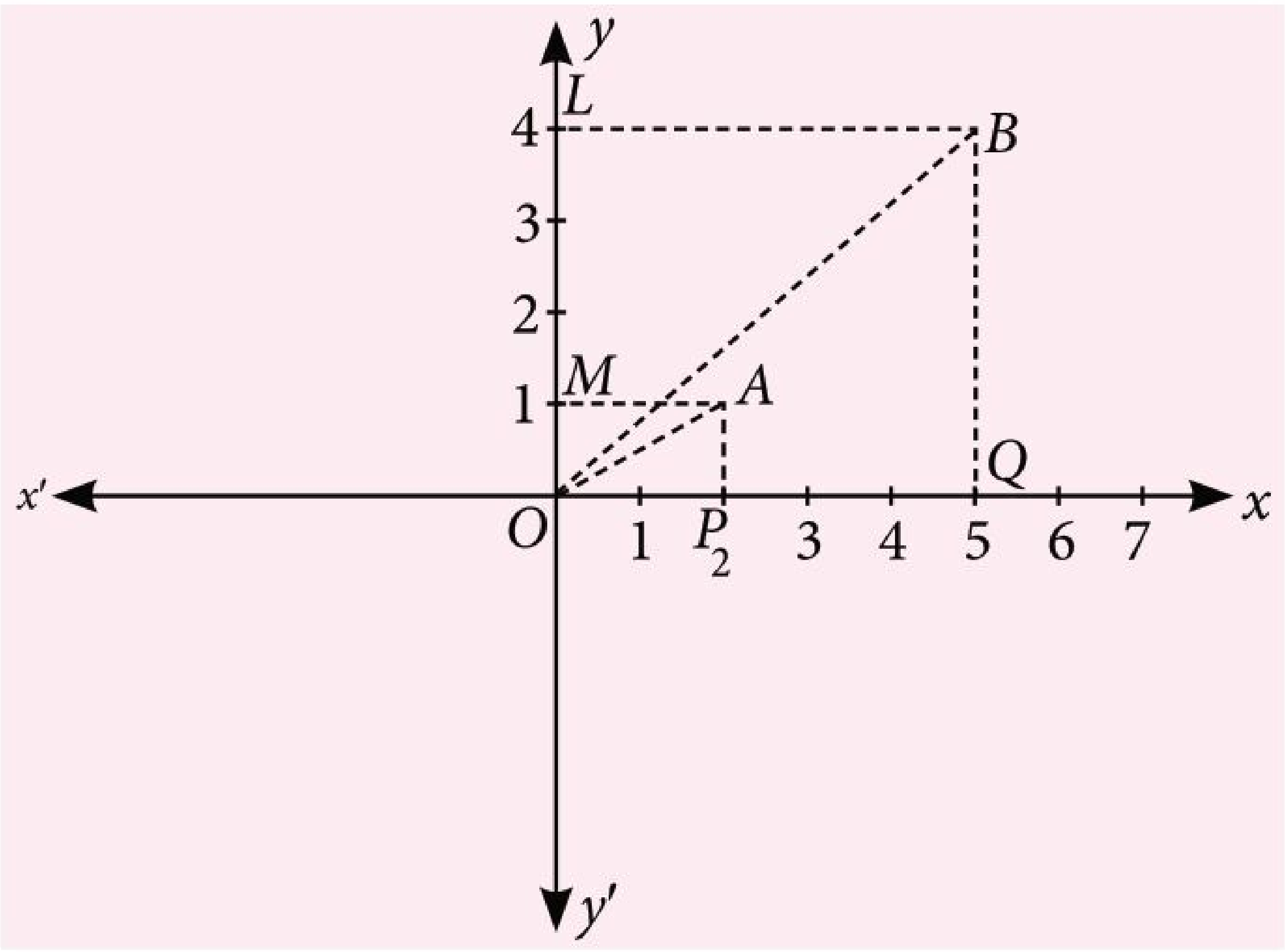
(ii) Modulus of A is

(a) 2 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) $2\sqrt{2}$

(iii) Lengths representing modulus of the points A and B respectively are

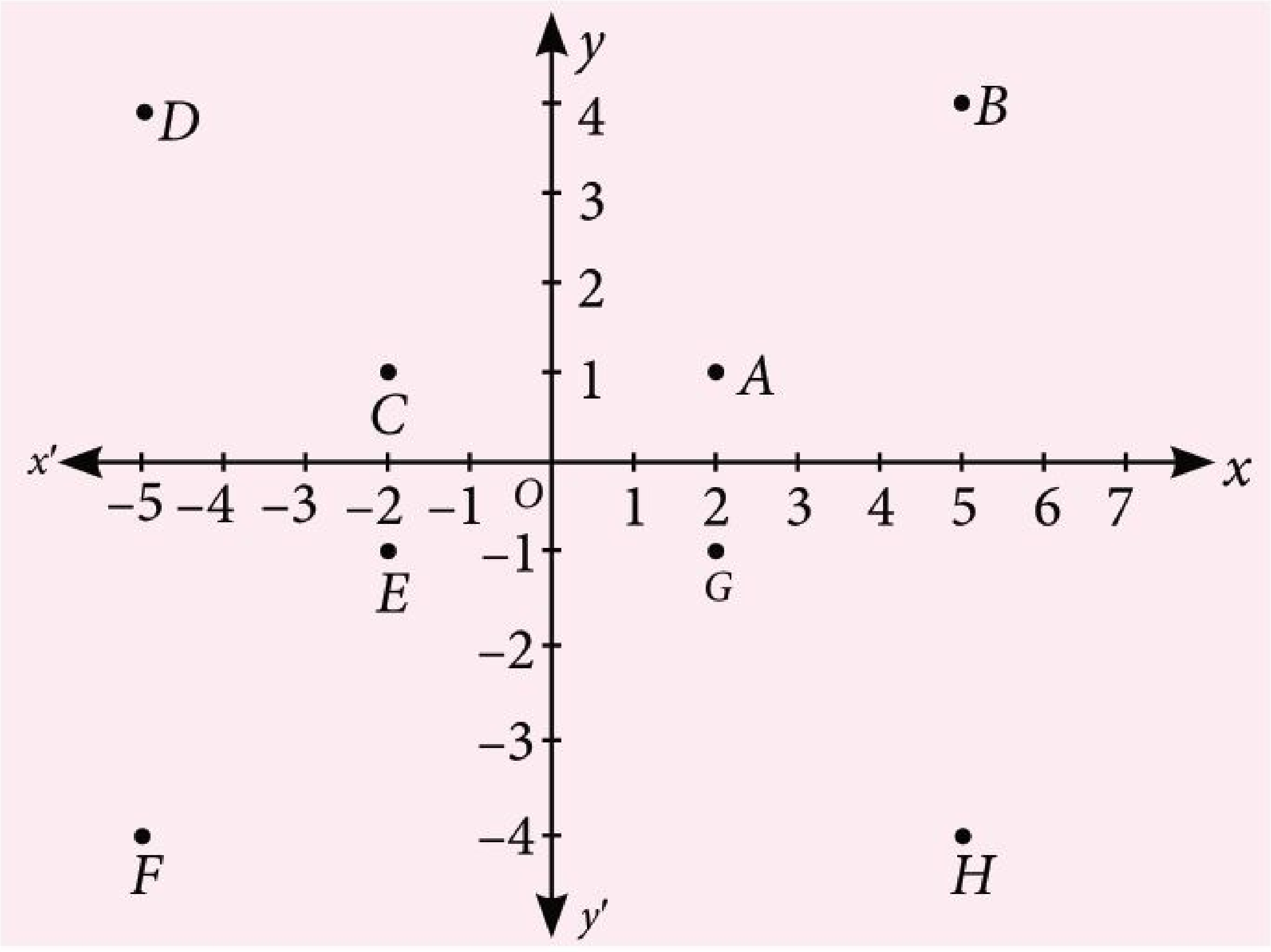
(a) PA, QB (b) MA, LB

(c) OA, OB (d) AB, OA



(iv) Conjugates of the complex numbers at the points A and B are respectively represented by the points.

(a) C, H (b) G, H (c) E, F (d) C, D



(v) Product of the complex numbers A and B is

(a) $6 + 13i$ (b) $13 + 6i$

(c) $6 - 13i$ (d) $14 + 13i$

PART - B

SECTION - III

19. Find the value of $\sin(40° + \theta)\cos(10° + \theta) - \cos(40° + \theta)\sin(10° + \theta)$.

20. Find the value of $\dfrac{1-\tan^2 15°}{1+\tan^2 15°}$.

21. If $\tan A = \dfrac{1}{2}$, $\tan B = \dfrac{1}{3}$, then what is the value of $\tan(2A + B)$.

22. Find the modulus of $\dfrac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$.

23. Find the multiplicative inverse of $\dfrac{3+4i}{4-5i}$.

OR

Find the modulus of the complex number

$$z = \frac{(1-i\sqrt{3})(\cos\theta + i\sin\theta)}{2(1-i)(\cos\theta - i\sin\theta)}.$$

24. Prove that :

$$\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}.$$

25. If $5\sin x = 3$, then find the value of $\dfrac{\sec x - \tan x}{\sec x + \tan x}$.

OR

Find the value of $2\sin^2\dfrac{3\pi}{4} + 2\cos^2\dfrac{3\pi}{4} - 2\tan^2\dfrac{3\pi}{4}$.

26. Find the solution of $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$ over complex numbers.

27. Find the conjugate, modulus and argument of $\sqrt{2} - \sqrt{2}i$.

28. If $iz^3 + z^2 - z + i = 0$, then show that $|z| = 1$.

OR

Find the value of $\dfrac{i^{4n+1} - i^{4n-1}}{2}$.

SECTION - IV

29. If $(1 + 2i)(2 + 3i)(3 + 4i) = x + iy$, then prove that $x^2 + y^2 = 1625$.

30. Find the multiplicative inverse of $\sqrt{5} + 3i$.

OR

If $z_1 = 2 - i$, $z_2 = 1 + i$, then find $\left|\dfrac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$.

31. Prove that:

$\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

32. Prove that $\cos 20° \cos 40° \cos 80° = \dfrac{1}{8}$.

33. Prove that : $\dfrac{\sec 8x - 1}{\sec 4x - 1} = \dfrac{\tan 8x}{\tan 2x}$

OR

Prove that : $\dfrac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

34. If the value of

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

is equal to k^2, then, find the value of k.

35. If $\dfrac{\sin(\theta + A)}{\sin(\theta + B)} = \sqrt{\dfrac{\sin 2A}{\sin 2B}}$ then find $\tan^2\theta$.

SECTION - V

36. If 3 sin θ + 5 cos θ = 5, then find the value of 5 sin θ – 3 cos θ.

OR

If x and y are acute angles such that $\sin x+\sin y=\frac{3}{4}$ and $\cos x+\cos y=\frac{3}{2}$, then find the value of $\sin(x+y)$.

37. Show that $\left|\frac{z-2}{z-3}\right|=2$ represents a circle. Find its centre and radius.

OR

If $|z + 1| = z + 2(1 + i)$, then find z.

38. If the real part of $\frac{\bar{z}+2}{\bar{z}-1}$ is 4, then show that the locus of the point representing z in the complex plane is a circle.

OR

If $p+iq=\frac{(a+i)^2}{2a-i}$, then show that $p^2+q^2=\frac{(a^2+1)^2}{4a^2+1}$.

SOLUTIONS

1. Since $\tan\theta=-\frac{4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin\theta=\frac{4}{5}$ if θ lies in the second quadrant or $\sin\theta=-\frac{4}{5}$, if θ lies in the fourth quadrant.

2. We have, $\cos x=\frac{4}{5}, x\in\left[0,\frac{\pi}{2}\right]$

$$\cos x=2\cos^2\frac{x}{2}-1 \Rightarrow 2\cos^2\frac{x}{2}=\frac{9}{5}$$

$$\Rightarrow \cos^2\frac{x}{2}=\frac{9}{10} \Rightarrow \cos\frac{x}{2}=\pm\frac{3}{\sqrt{10}}$$

$$\therefore\quad \cos\frac{x}{2}=\frac{3}{\sqrt{10}}$$

$$\left[\because\ \cos\frac{x}{2}\text{ is positive in I quadrant }\left(0<\frac{x}{2}<\frac{\pi}{4}\right)\right]$$

OR

The angle traced by the hour hand in 12 hrs = 360°

∴ The angle traced by the hour hand in 3 hrs 30 mins

i.e., $\frac{7}{2}\text{hr}=\left(\frac{360}{12}\times\frac{7}{2}\right)^\circ=105^\circ$

The angle traced by the minute hand in 60 mins = 360°

∴ The angle traced by the minute hand in 30 mins

$$=\left(\frac{360}{60}\times 30\right)^\circ=180^\circ$$

Hence, the required angle between two hands

= 180° – 105° = 75°

Required angle in radians $=\left(75\times\frac{\pi}{180}\right)=\frac{5\pi}{12}$ rad

3. We have, $(-5i)\left(\frac{1}{8}i\right)=\frac{-5}{8}i^2=\frac{-5}{8}(-1)=\frac{5}{8}=\frac{5}{8}+0i$

4. Since $x^2 + y^2 = (x + iy)(x - iy)$

$$\therefore\quad (p+iq)=\left(\frac{x^2+y^2}{x+iy}\right)i=(x-iy)i=y+ix$$

$$\therefore\quad p=y, q=x$$

OR

We have, $(-\sqrt{-1})^{4n+3}=(-i)^{4n+3}=(-i)^{4n}(-i)^3$

$=\{(-i)^4\}^n(-i^3)=1\times -i^3=i$

5. cos1° cos2° cos3° … cos90° … cos179°

= cos1° cos2° cos3° … 0 … cos179° = 0

6. Given, $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$

$=\frac{\cot\theta-1}{1+\cot\theta}\times\frac{1+\cot\theta}{\cot\theta-1}=1$

OR

We have, $\tan\alpha=\frac{1}{7}$

Now, $\cos 2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\frac{1-\frac{1}{49}}{1+\frac{1}{49}}=\frac{48}{50}$

7. Reciprocal of $z=\frac{\bar{z}}{|z|^2}$

Therefore, reciprocal of $3+\sqrt{7}i=\frac{3-\sqrt{7}i}{16}=\frac{3}{16}-\frac{\sqrt{7}i}{16}$

8. $\sec\frac{2\pi}{3}+\operatorname{cosec}\frac{5\pi}{6}=\sec\left(\pi-\frac{\pi}{3}\right)+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)$

$=-\sec\frac{\pi}{3}+\operatorname{cosec}\frac{\pi}{6}=-2+2=0$

OR

We have, sin50° – sin70° + sin10°

= (sin50° + sin10°) – sin70°

$=2\sin\left(\frac{50+10}{2}\right)^\circ\cos\left(\frac{50-10}{2}\right)^\circ-\sin 70^\circ$

= 2sin30° cos20° – sin70°

= cos(90 – 70)° – sin70° = sin70° – sin70° = 0

9. $\sqrt{-3}=i\sqrt{3}, \sqrt{-6}=i\sqrt{6}$

So, $\sqrt{(-3)}\sqrt{(-6)}=i^2 3\sqrt{2}=-3\sqrt{2}+i(0)$

10. $i^{-35}=\frac{1}{i^{35}}=\frac{1}{(i^2)^{17}i}=\frac{1}{-i}\times\frac{i}{i}=\frac{i}{-i^2}=i$

11. $\sqrt{3}$ cosec 20° – sec 20° = $\frac{\sqrt{3}}{\sin 20^\circ}-\frac{1}{\cos 20^\circ}$

$$=\frac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}=4\left(\frac{\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ}{2\sin 20^\circ\cos 20^\circ}\right)$$

$$=4\left(\frac{\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ}{\sin 40^\circ}\right)$$

$$=4\left(\frac{\sin(60^\circ-20^\circ)}{\sin 40^\circ}\right)=4$$

12. We have, 2sin15° cos75° = sin(15° + 75°) + sin(15° – 75°)

$= \sin 90^\circ - \sin 60^\circ = \frac{2-\sqrt{3}}{2}$

OR

$240^\circ=\frac{\pi}{180}\times 240$ radian $=\frac{4\pi}{3}$ radian

13. $40^\circ 20'=\left(40\frac{20}{60}\right)^\circ=\left(\frac{121}{3}\right)^\circ$

Now, $1^\circ=\left(\frac{\pi}{180}\right)^c\Rightarrow\left(\frac{121}{3}\right)^\circ=\left(\frac{121}{3}\times\frac{\pi}{180}\right)^c=\left(\frac{121\pi}{540}\right)^c$

14. Given, $\sin\theta+\operatorname{cosec}\theta=2$

On squaring both sides, we get

$\sin^2\theta+\operatorname{cosec}^2\theta+2=4\quad\therefore\ \sin^2\theta+\operatorname{cosec}^2\theta=2$

15. The first $2n$ terms will cancel in pairs and the last term is $(i^2)^{2n+1}=i^{4n}\cdot i^2=-1$

16. We have, $(1+i)^{2n}=(1-i)^{2n}\Rightarrow\left(\frac{1+i}{1-i}\right)^{2n}=1$

$\Rightarrow\ (i)^{2n}=1$, which is possible if $n=2$ $\quad(\because\ i^4=1)$

17. (i) → (d), (ii) → (c), (iii) → (b), (iv) → (a), (v) → (b)

18. (i) → (d), (ii) → (b), (iii) → (c), (iv) → (b), (v) → (a)

19. We have,

$\sin(40^\circ+\theta)\cdot\cos(10^\circ+\theta)-\cos(40^\circ+\theta)\cdot\sin(10^\circ+\theta)$

$=\sin\{(40^\circ+\theta)-(10^\circ+\theta)\}=\sin 30^\circ=\frac{1}{2}$

20. $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}=\frac{\cos^2 15^\circ-\sin^2 15^\circ}{\cos^2 15^\circ+\sin^2 15^\circ}$

$=\frac{\cos(2\times 15^\circ)}{1}=\cos 30^\circ=\frac{\sqrt{3}}{2}$

21. $\tan A=\frac{1}{2},\ \tan B=\frac{1}{3}$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}=\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}=\frac{4}{3}$$

$$\therefore\ \tan(2A+B)=\frac{\tan 2A+\tan B}{1-\tan 2A\tan B}=\frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\cdot\frac{1}{3}}=1$$

22. $\left|\frac{(1+i\sqrt{3})(2+2i)}{\sqrt{3}-i}\right|=\frac{|1+i\sqrt{3}||2+2i|}{|\sqrt{3}-i|}=\frac{2\times 2\sqrt{2}}{2}=2\sqrt{2}$

23. Let $z=\frac{3+4i}{4-5i}\times\frac{4+5i}{4+5i}=-\frac{8}{41}+\frac{31}{41}i$

Then $\bar{z}=-\frac{8}{41}-\frac{31}{41}i$

and $|z|=\sqrt{\left(-\frac{8}{41}\right)^2+\left(\frac{31}{41}\right)^2}=\frac{5}{\sqrt{41}}$

$\therefore$ Multiplicative inverse of z

$$=\frac{\bar{z}}{|z|^2}=\frac{-\frac{8}{41}-\frac{31}{41}i}{\frac{25}{41}}=-\frac{8}{25}-\frac{31}{25}i$$

OR

$|\cos\theta+i\sin\theta|=|\cos\theta-i\sin\theta|=1$

$\therefore\ |z|=\frac{|1-i\sqrt{3}|}{2|1-i|}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$

24. L.H.S. = $\tan 4x=\tan(2(2x))$

$$=\frac{2\tan 2x}{1-\tan^2 2x}=\frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$=\frac{4\tan x(1-\tan^2 x)^2}{(1-\tan^2 x)[(1-\tan^2 x)^2-4\tan^2 x]}$$

$$=\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}=\text{R.H.S.}$$

25. We have, $5\sin x=3\Rightarrow\sin x=\frac{3}{5}$

Since, $\sin^2 x+\cos^2 x=1$

$\Rightarrow\ \cos^2 x=1-\sin^2 x=1-\frac{9}{25}=\frac{16}{25}.$

$\Rightarrow\ \cos x=\pm\frac{4}{5}$

$\therefore\ \sec x=\pm\frac{5}{4}$ and $\tan x=\pm\left(\frac{3}{5}\times\frac{5}{4}\right)=\pm\frac{3}{4}$

Hence, $\dfrac{\sec x - \tan x}{\sec x + \tan x} = \dfrac{\frac{5}{4}-\frac{3}{4}}{\frac{5}{4}+\frac{3}{4}} = \dfrac{2}{4}\times\dfrac{4}{8} = \dfrac{1}{4}$

OR

Consider, $2\sin^2\dfrac{3\pi}{4}+2\cos^2\dfrac{3\pi}{4}-2\tan^2\dfrac{3\pi}{4}$ ...(i)

$\sin\dfrac{3\pi}{4} = \sin 135° = \sin(180° - 45°) = \sin 45° = \dfrac{1}{\sqrt{2}}$

$\cos\dfrac{3\pi}{4} = \cos 135° = \cos(180° - 45°) = -\cos 45° = -\dfrac{1}{\sqrt{2}}$

$\tan\dfrac{3\pi}{4} = \tan 135° = \tan(180° - 45°) = -\tan 45° = -1$

Substituting in (i), we get

$2\left(\dfrac{1}{\sqrt{2}}\right)^2 + 2\left(-\dfrac{1}{\sqrt{2}}\right)^2 - 2(-1)^2$

$= 2\times\dfrac{1}{2} + 2\times\dfrac{1}{2} - 2\times 1 = 1 + 1 - 2 = 0$

26. Consider, $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

$D = (-\sqrt{2})^2 - 4\times\sqrt{3}\times 3\sqrt{3} = 2 - 36 = -34 < 0$

$\therefore\ x = \dfrac{-(-\sqrt{2}) \pm \sqrt{-34}}{2\sqrt{3}} = \dfrac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$

27. Let $z = \sqrt{2} - \sqrt{2}i$

Here, $x = \sqrt{2}, y = -\sqrt{2}$

$\bar{z} = \sqrt{2} + \sqrt{2}i$

and $|z| = |\sqrt{2} - \sqrt{2}i| = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{2+2} = 2$

Argument of $z : \tan\theta = \left|\dfrac{y}{x}\right| = \left|\dfrac{-\sqrt{2}}{\sqrt{2}}\right| = 1 = \tan 45°$

$\therefore\quad \theta = \dfrac{\pi}{4}$

$\because\quad x = \sqrt{2} > 0$ and $y = -\sqrt{2} < 0$

Hence z lies in the fourth quadrant.

$\therefore\quad \arg z = 2\pi - \dfrac{\pi}{4} = \dfrac{7\pi}{4}$

28. Given, $iz^3 + z^2 - z + i = 0$

$\Rightarrow\ iz^2(z - i) - (z - i) = 0$

$\Rightarrow\ (z - i)(iz^2 - 1) = 0 \Rightarrow z = i$ or $z^2 = \dfrac{1}{i} = -i$

Now, $z = i \Rightarrow |z| = |i| = 1$

and $z^2 = -i \Rightarrow |z^2| = |-i| \Rightarrow |z|^2 = 1 \Rightarrow |z| = 1$

Thus in both cases $|z| = 1$.

OR

We have, $\dfrac{i^{4n+1} - i^{4n-1}}{2}$

$= \dfrac{i^{4n}\cdot i - i^{4n}\cdot i^{(-1)}}{2} = \dfrac{(i - i^{-1})}{2}$ $[\because\ i^4 = 1]$

$= \dfrac{1}{2}\left(i - \dfrac{1}{i}\right) = \dfrac{1}{2}\left(i - \dfrac{1\times i}{i\times i}\right) = \dfrac{1}{2}(i + i) = i$

29. We have, $x + iy = (1 + 2i)(2 + 3i)(3 + 4i)$

$= (2 + 3i + 4i + 6i^2)(3 + 4i) = (-4 + 7i)(3 + 4i)$

$= -12 - 16i + 21i + 28i^2 = -40 + 5i$

Equating the real and the imaginary parts, we get

$x = -40, y = 5$

Now, $x^2 + y^2 = (-40)^2 + (5)^2 = 1600 + 25 = 1625$

30. Let $z = \sqrt{5} + 3i$

$\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + (3)^2 = 14$

$z^{-1} = \dfrac{\bar{z}}{|z|^2} = \dfrac{\sqrt{5} - 3i}{14} = \dfrac{\sqrt{5}}{14} - \dfrac{3}{14}i$

OR

Given, $z_1 = 2 - i$ and $z_2 = 1 + i$

Now, $\left|\dfrac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = \left|\dfrac{2 - i + 1 + i + 1}{2 - i - 1 - i + i}\right|$

$= \left|\dfrac{4}{1 - i}\right| = \left|\dfrac{4(1 + i)}{(1 - i)(1 + i)}\right| = \left|\dfrac{4 + 4i}{1 - i^2}\right| = \left|\dfrac{4 + 4i}{2}\right|$

$= |2 + 2i| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

31. L.H.S. $= \cos 6x = 2\cos^2 3x - 1$

$= 2(4\cos^3 x - 3\cos x)^2 - 1$

$= 2(16\cos^6 x + 9\cos^2 x - 24\cos^4 x) - 1$

$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1 =$ R.H.S.

32. L.H.S. $= \cos 20° \cos 40° \cos 80°$

$= \cos 20° \cos(60° - 20°) \cos(60° + 20°)$

$= \cos 20° (\cos 60° \cos 20° + \sin 60° \sin 20°)$
$(\cos 60° \cos 20° - \sin 60° \sin 20°)$

$= \cos 20°\left(\dfrac{\cos 20°}{2} + \dfrac{\sqrt{3}\sin 20°}{2}\right)\left(\dfrac{\cos 20°}{2} - \dfrac{\sqrt{3}}{2}\sin 20°\right)$

$= \cos 20°\left(\dfrac{\cos^2 20°}{4} - \dfrac{3}{4}\sin^2 20°\right)$

$= \cos 20°\left(\dfrac{\cos^2 20°}{4} - \dfrac{3}{4}(1 - \cos^2 20°)\right)$

$= \cos 20°\left(\dfrac{\cos^2 20°}{4} + \dfrac{3\cos^2 20°}{4} - \dfrac{3}{4}\right)$

$$= \cos 20^\circ\left(\cos^2 20^\circ - \frac{3}{4}\right)$$

$$= \frac{4\cos^3 20^\circ - 3\cos 20^\circ}{4} = \frac{\cos 60^\circ}{4} = \frac{1}{2\cdot 4} = \frac{1}{8} = \text{R.H.S.}$$

33. $$\text{L.H.S.} = \frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\frac{1}{\cos 8x} - 1}{\frac{1}{\cos 4x} - 1}$$

$$= \frac{(1-\cos 8x)\cos 4x}{(1-\cos 4x)\cos 8x} = \frac{2\sin^2 4x \cos 4x}{2\sin^2 2x \cos 8x}$$

$$= \frac{\sin 4x(2\sin 4x \cos 4x)}{2\cos 8x \sin^2 2x} = \frac{\sin 4x \sin 8x}{2\cos 8x \sin^2 2x}$$

$$= \frac{2\sin 2x \cos 2x \sin 8x}{2\cos 8x \sin^2 2x} = \frac{\tan 8x}{\tan 2x} = \text{R.H.S.}$$

OR

$$\text{L.H.S.} = \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x}$$

$$= \frac{2\sin 3x \cos 2x - 2\sin 3x}{-2\sin 3x \sin 2x} = \frac{\sin 3x(\cos 2x - 1)}{-\sin 3x \sin 2x}$$

$$= \frac{1-\cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x \cos x} = \tan x = \text{R.H.S.}$$

34. $$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

$$= \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{\pi}{2}\sin\frac{5\pi}{14}\sin\frac{3\pi}{14}\sin\frac{\pi}{14}$$

$$= \sin^2\frac{\pi}{14}\sin^2\frac{3\pi}{14}\sin^2\frac{5\pi}{14}\times 1 = \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$$= \left(\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\right)$$

$$= \left(\cos\frac{6\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\right)^2 = \left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\right)^2$$

$$= \left(-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right)^2 = \left(\frac{\sin 2^3\frac{\pi}{7}}{2^3\sin\frac{\pi}{7}}\right)^2$$

$$= \left(\frac{\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}}\right)^2 = \frac{\left(-\sin\frac{\pi}{7}\right)^2}{\left(8\sin\frac{\pi}{7}\right)^2} = \frac{1}{64} = k^2 \Rightarrow |k| = \frac{1}{8}$$

35. $$\frac{\sin(\theta+A)}{\sin(\theta+B)} = \sqrt{\frac{\sin 2A}{\sin 2B}}$$

Squaring on both sides, $\dfrac{\sin^2(\theta+A)}{\sin^2(\theta+B)} = \dfrac{\sin 2A}{\sin 2B}$

$$\Rightarrow \frac{\sin 2A}{\sin 2B} = \frac{1-\cos 2(\theta+A)}{1-\cos 2(\theta+B)} \quad [\because 1 - \cos 2\phi = 2\sin^2\phi]$$

$\Rightarrow$ $\sin 2A\,(1 - \cos 2(\theta + B)) = \sin 2B\,(1 - \cos 2(\theta + A))$

$\Rightarrow$ $\sin 2A - \sin 2A\,(\cos 2(\theta + B)) = \sin 2B - \sin 2B\,(\cos 2(\theta + A))$

Multiplying by 2 on both sides, we get

$2(\sin 2A - \sin 2B) = 2(\sin 2A\,(\cos 2(\theta + B)) - \sin 2B(\cos 2(\theta + A)))$

$\Rightarrow$ $4\cos(A + B)\sin(A - B) = \sin 2(A - \theta - B) + \sin 2(\theta + A - B)$

$= 4\cos(A + B)\sin(A - B) = 2\sin 2(A - B)\cos 2\theta$

$= 4\sin(A - B)\cos(A - B)\cos 2\theta$

$$\Rightarrow \cos 2\theta = \frac{\cos(A+B)}{\cos(A-B)}$$

Using $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$, we get

$$\tan^2\theta = \frac{\cos(A-B)-\cos(A+B)}{\cos(A-B)+\cos(A+B)}$$

$\tan^2\theta = \tan A\cdot\tan B$

[Using formulae of $\cos A - \cos B$ and $\cos A + \cos B$]

36. Given equation is $3\sin\theta + 5\cos\theta = 5$

$\Rightarrow$ $3\sin\theta = 5(1 - \cos\theta)$

$\Rightarrow$ $9\sin^2\theta = 25(1 - \cos\theta)^2$ [By squaring]

$\Rightarrow$ $9\sin^2\theta = 25(1 + \cos^2\theta - 2\cos\theta)$

$\Rightarrow$ $9\sin^2\theta = 25 + 25\cos^2\theta - 25 \times 2\cos\theta$

$\Rightarrow$ $9(1 - \cos^2\theta) = 25 + 25\cos^2\theta - 50\cos\theta$

[$\because \sin^2\theta = 1 - \cos^2\theta$]

$\Rightarrow$ $34\cos^2\theta - 50\cos\theta + 16 = 0$

After solving quadratic equation, we get

$\cos\theta = 1$ or $\cos\theta = \dfrac{8}{17}$ $\Rightarrow$ $\sin\theta = 0$ or $\sin\theta = \dfrac{15}{17}$

Hence if $\cos\theta = 1$, we have

$5\sin\theta - 3\cos\theta = 5 \times 0 - 3 \times 1 = -3$

and when $\cos\theta = \dfrac{8}{17}$, we have

$$5\sin\theta - 3\cos\theta = 5\times\frac{15}{17} - 3\times\frac{8}{17} = \frac{(75-24)}{17} = \frac{51}{17} = 3$$

OR

We have, $\sin x + \sin y = \dfrac{3}{4}$ ...(i)

and $\cos x + \cos y = \dfrac{3}{2}$ ...(ii)

Dividing (i) by (ii), we get

$$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{3/4}{3/2} = \frac{1}{2}$$

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{1}{2} \Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

Now, $\sin(x+y) = \dfrac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)} = \dfrac{2\times\frac{1}{2}}{1+\frac{1}{4}} = \dfrac{1}{\frac{5}{4}} = \dfrac{4}{5} = 0.8.$

37. We have, $\left|\dfrac{z-2}{z-3}\right| = 2$

Let $z = x + iy$, then $\left|\dfrac{x+iy-2}{x+iy-3}\right| = 2$

$\Rightarrow$ $|x - 2 + iy| = 2|x - 3 + iy|$

$\Rightarrow$ $\sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$\Rightarrow$ $x^2 - 4x + 4 + y^2 = 4(x^2 - 6x + 9 + y^2)$

$\Rightarrow$ $3x^2 + 3y^2 - 20x + 32 = 0 \Rightarrow x^2 + y^2 - \dfrac{20}{3}x + \dfrac{32}{3} = 0$

$\Rightarrow$ $\left(x - \dfrac{10}{3}\right)^2 + y^2 + \dfrac{32}{3} - \dfrac{100}{9} = 0$

$\Rightarrow$ $\left(x - \dfrac{10}{3}\right)^2 + (y-0)^2 = \dfrac{4}{9}$, which is equation of circle, whose centre is $\left(\dfrac{10}{3}, 0\right)$ and radius $= \dfrac{2}{3}$ unit.

OR

We have, $|z + 1| = z + 2\ (1 + i)$

Putting $z = x + iy$, we get

$\Rightarrow$ $|x + iy + 1| = x + iy + 2(1 + i)$

$\Rightarrow$ $|x + 1 + iy| = (x + 2) + i(y + 2)$

$\Rightarrow$ $\sqrt{(x+1)^2 + y^2} = (x+2) + i(y+2)$

$\left[\because |z| = \sqrt{x^2 + y^2}\right]$

Comparing real and imaginary parts, we get

$\sqrt{(x+1)^2 + y^2} = x + 2 \Rightarrow (x+1)^2 + y^2 = (x+2)^2$ ...(i)

and $0 = y + 2 \Rightarrow y = -2$ ...(ii)

Substituting the value of $y = -2$ in (i), we get

$(x + 1)^2 + (-2)^2 = (x + 2)^2$

$\Rightarrow$ $x^2 + 2x + 1 + 4 = x^2 + 4x + 4$

$\Rightarrow$ $2x + 1 = 4x \Rightarrow x = 1/2$

Hence, $z = x + iy = \dfrac{1}{2} - 2i$

38. Let $z = x + iy$, $\bar{z} = x - iy$

$\therefore$ $\dfrac{\bar{z}+2}{\bar{z}-1} = \dfrac{x-iy+2}{x-iy-1}$

$= \dfrac{[(x+2)-iy][(x-1)+iy]}{[(x-1)-iy][(x-1)+iy]}$

$= \dfrac{(x-1)(x+2) - iy(x-1) + iy(x+2) + y^2}{(x-1)^2 + y^2}$

$= \dfrac{(x-1)(x+2) + y^2 + i[(x+2)y - (x-1)y]}{(x-1)^2 + y^2}$

Since $\text{Re}\left(\dfrac{\bar{z}+2}{\bar{z}-1}\right) = 4$

$\Rightarrow$ $\dfrac{(x-1)(x+2) + y^2}{(x-1)^2 + y^2} = 4$

$\Rightarrow$ $x^2 - x + 2x - 2 + y^2 = 4(x^2 - 2x + 1 + y^2)$

$\Rightarrow$ $3x^2 + 3y^2 - 9x + 6 = 0,$

which is an equation of a circle.

OR

We have, $p + iq = \dfrac{(a+i)^2}{2a-i}$ ...(i)

Taking conjugate on both sides, we get

$p - iq = \dfrac{(a-i)^2}{2a+i}$ ...(ii)

Multiplying (i) and (ii), we get

$(p+iq)(p-iq) = \dfrac{(a+i)^2}{(2a-i)} \times \dfrac{(a-i)^2}{(2a+i)}$

$\Rightarrow$ $p^2 - (iq)^2 = \dfrac{[(a+i)(a-i)]^2}{(2a-i)(2a+i)}$

$\Rightarrow$ $p^2 + q^2 = \dfrac{(a^2+1)^2}{4a^2+1}$

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 2 : Complex Numbers and Quadratic Equations

Time Taken : 60 Min.

Only One Option Correct Type

1. If $z = x + iy$ and $x^2 + y^2 = 16$, then the range of $||x| - |y||$ is

(a) $[0, 4]$ (b) $[0, 2]$
(c) $[2, 4]$ (d) None of these

2. If $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then $z_1^2 + z_2^2 + z_3^2$ equals

(a) -3 (b) $-\frac{1}{3}$ (c) 0 (d) 3

3. If $\frac{3}{2+\cos\theta+i\sin\theta} = a+ib$, then $(a - 2)^2 + b^2$ equals

(a) 0 (b) 1 (c) -1 (d) 2

4. If $a = \cos\theta + i\sin\theta$ and $b = \cos\phi + i\sin\phi$, then

(a) $\cos(\theta+\phi) = \left(ab - \frac{1}{ab}\right)$

(b) $\cos(\theta+\phi) = \frac{1}{2}\left(\frac{a}{b} + \frac{b}{a}\right)$

(c) $\cos(\theta+\phi) = \frac{1}{2}\left(ab + \frac{1}{ab}\right)$

(d) $\cos(\theta+\phi) = \left(\frac{a^2-b^2}{ab}\right)$

5. If $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(a) Real and distinct (b) Real and equal
(c) Imaginary (d) None of these

6. Given that $\tan A$ and $\tan B$ are the roots of $x^2 - px + q = 0$. Then the value of $\sin^2(A + B)$ is

(a) $\frac{p^2}{p^2+(1-q)^2}$ (b) $\frac{p^2}{p^2+q^2}$

(c) $\frac{q^2}{p^2+(1-q)^2}$ (d) $\frac{p^2}{(1+q)^2}$

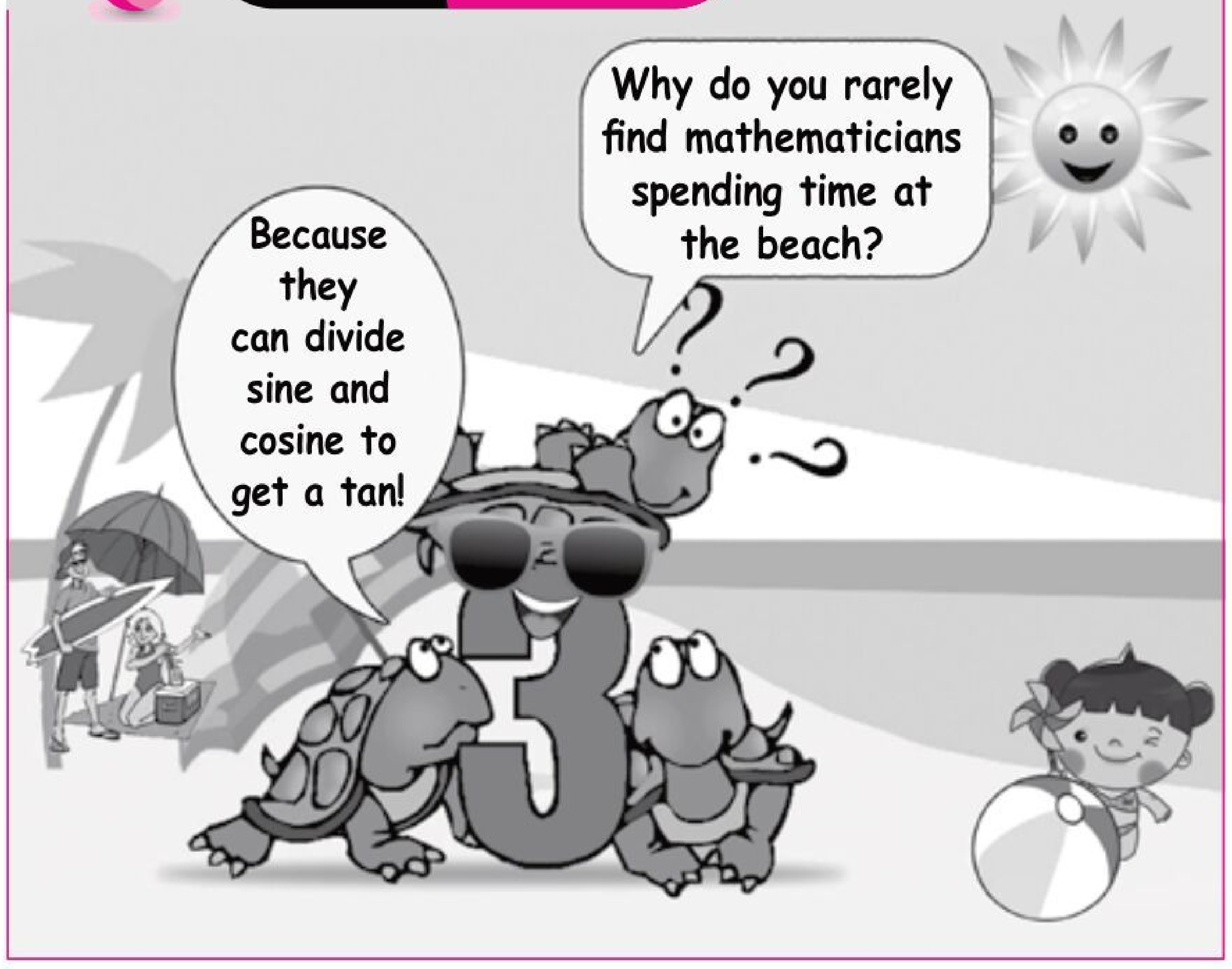
More Than One Option(s) Correct Type

7. If p, q, r are positive real numbers and are in A.P., the roots of the quadratic equation $px^2 + qx + r = 0$ are real for

(a) $\left|\frac{r}{p} - 7\right| > 4\sqrt{3}$ (b) $\left|\frac{r}{p} - 7\right| = 4\sqrt{3}$

(c) All p and all r (d) No p and no r

COMIC CAPSULE


Why do you rarely find mathematicians spending time at the beach?
Because they can divide sine and cosine to get a tan!
3

8. If $y = \log_{7-a}(4x^2 + 2x + a + 3)$ is defined $\forall\ x \in R$, then possible integral value(s) of a is(are)
(a) -3 (b) -2 (c) 4 (d) 5

9. If $c \neq 0$ and the equation $p/(2x) = a/(x + c) + b/(x - c)$ has two equal roots, then p can be
(a) $(\sqrt{a}-\sqrt{b})^2$ (b) $(\sqrt{a}+\sqrt{b})^2$
(c) $a + b$ (d) $a - b$

10. The equation $x^{[\log_3 x)^2-(9/2)\log_3 x+5]} = 3\sqrt{3}$ has
(a) at least one real solution
(b) exactly three real solutions
(c) exactly one irrational solution
(d) complex roots

11. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then
(a) $\frac{x}{y} + \frac{y}{x} = 2\cos(\theta - \phi)$
(b) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$
(c) $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$
(d) $xy + \frac{1}{xy} = 2\cos(\theta + \phi)$

12. If $x \in R$, then range of $\frac{x^2 - 2x + 3}{x^2 - 2x - 8}$ is
(a) $\left[\frac{-2}{9}, 1\right)$ (b) $\left[\frac{-2}{9}, 1\right]$
(c) $\left(-\infty, -\frac{2}{9}\right] \cup (1, \infty)$ (d) None of these

13. The common roots of the equations $z^3 + (1 + i)z^2 + (1 + i)z + i = 0$, (where $i = \sqrt{-1}$) and $z^{1993} + z^{1994} + 1 = 0$ are
(a) 1 (b) ω
(c) ω^2 (d) $-\omega^{981}$

Comprehension Type

Consider the equation $x^4 - \lambda x^2 + 9 = 0$.

14. If the equation has four real and distinct roots, then λ lies in the interval
(a) $(-\infty, -6) \cup (6, \infty)$ (b) $(0, \infty)$
(c) $(6, \infty)$ (d) $(-\infty, -6)$

15. If the equation has no real root, then λ lies in the interval
(a) $(-\infty, 0)$ (b) $(-\infty, 6)$ (c) $(6, \infty)$ (d) $(0, \infty)$

Matrix Match Type

16. Match the following:

	Column-I	Column-II	
P.	If $(3a - 2ib)(2 + i)^2 = 10(1 + i)$, then reciprocal of $(a - 3b)$ is	1.	3
Q.	If $z + z^3 = 0$, then Re(z) is equal to	2.	2
R.	If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root, then value of $4\lvert\alpha + \bar{\alpha}\rvert$ is	3.	4
S.	If $x, y \in R$, and $2x^2 + 6xy + 5y^2 = 1$, then maximum value of y^2 is	4.	0

	P	Q	R	S
(a)	1	4	3	2
(b)	2	1	3	4
(c)	3	1	4	2
(d)	3	4	1	2

Numerical Answer Type

17. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + 2z_2 + 3z_3| = 6$, then find the value of $|z_2z_3 + 8z_3z_1 + 27z_1z_2|$ _____.

18. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and $a \neq 0$, then $\frac{a^3 + b^3 + c^3}{abc} =$ _____.

19. If $|z| = \max\{|z - 1|, |z + 1|\}$, then value of $|z + \bar{z}|$ is _____.

20. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is _____.

◆ ◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Series-2

Matrices and Determinants

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-*I *and* II.
2. *Section-I comprises of* 16 *very short answer type questions.*
3. *Section-II contains* 2 *case study-based questions.*

Part - B :

1. *It consists of three Sections-*III, IV *and* V.
2. *Section-*III *comprises of* 10 *questions of* 2 *marks each.*
3. *Section-*IV *comprises of* 7 *questions of* 3 *marks each.*
4. *Section-*V *comprises of* 3 *questions of* 5 *marks each.*
5. *Internal choice is provided in* 3 *questions of Section-*III, 2 *questions of Section-*IV *and* 3 *questions of Section-*V. *You have to attempt only one of the alternatives in all such questions.*

PART - A

SECTION - I

1. Find x, y, z and w respectively such that $\begin{bmatrix} x-y & 2z+w \\ 2x-y & 2x+w \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 12 & 15 \end{bmatrix}$.

2. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then find the value of α for which $A^2 = B$.

3. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find the values of a and b respectively.

OR

If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = i + j$, then find A.

4. Find the determinant of matrix $A = \begin{bmatrix} x & -7 \\ x & 5x+1 \end{bmatrix}$

5. Solve $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$.

6. If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = \frac{(i+2j)^2}{2}$, then find A.

7. If A is a square matrix of order 3 with $|A| = 9$, then write the value of $|2\cdot \text{adj } A|$.

OR

If for any 2×2 square matrix A, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

8. If $\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$, then find the value of $x - y$.

9. For a 2×3 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \dfrac{(i+2j)^2}{4}$, write the value of $a_{13} \times a_{23}$.

10. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of y.

OR

Find the determinant of the matrix

$$\begin{bmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{bmatrix}.$$

11. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then find the value of x.

12. Show that $A'A$ and AA' are both symmetric matrices for any matrix A.

13. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$,

then write the value of $|AB|$.

OR

If $A = [a_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ and $B = [b_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$,

then find the value of $a_{11}\, b_{11} + a_{22}\, b_{22}$.

14. Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

15. If the points $(2, -3)$, $(x, -1)$ and $(0, 4)$ are collinear, then find the value of x.

16. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then find the value of x.

OR

Find the minor and co-factor of the elements of the determinant $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$ and verify that

$a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$.

SECTION - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub part carries 1 mark.

17. Each triangular face of the Pyramid of Peace in Kazakhstan is made up of 25 smaller equilateral triangles as shown in the figure.

Using the above information and concept of determinants, answer the following questions.

(i) If the vertices of one of the smaller equilateral triangle are $(0, 0)$, $(3, \sqrt{3})$ and $(3, -\sqrt{3})$, then the area of such triangle is

(a) $\sqrt{3}$ sq. units
(b) $2\sqrt{3}$ sq. units
(c) $3\sqrt{3}$ sq. units
(d) none of these

(ii) The area of a face of the Pyramid is

(a) $25\sqrt{3}$ sq. units
(b) $50\sqrt{3}$ sq. units
(c) $75\sqrt{3}$ sq. units
(d) $35\sqrt{3}$ sq. units

(iii) The length of a altitude of a smaller equilateral triangle is

(a) 2 units
(b) 3 units
(c) $\sqrt{3}$ units
(d) 4 units

(iv) If (2, 4), (2, 6) are two vertices of a smaller equilateral triangle, then the third vertex will lie on the line represented by

(a) $x + y = 5$ (b) $x = 1 + \sqrt{3}$

(c) $x = 2 + \sqrt{3}$ (d) $2x + y = 5$

(v) Let $A(a, 0)$, $B(0, b)$ and $C(1, 1)$ be three points. If $\frac{1}{a} + \frac{1}{b} = 1$, then the three points are

(a) vertices of an equilateral triangle

(b) vertices of a right angled triangle

(c) collinear

(d) vertices of an isosceles triangle

18. In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

(i) If P represents the matrix of number of units of each type produced by factory A for both boys and girls, then P is given by

(a) $\begin{array}{ccc} & \text{Boys} & \text{Girls} \\ \text{I} & 85 & 50 \\ \text{II} & 65 & 55 \\ \text{III} & 72 & 80 \end{array}$ (b) $\begin{array}{cccc} & \text{I} & \text{II} & \text{III} \\ \text{Boys} & 50 & 55 & 80 \\ \text{Girls} & 85 & 65 & 72 \end{array}$

(c) $\begin{array}{cccc} & \text{I} & \text{II} & \text{III} \\ \text{Boys} & 80 & 75 & 90 \\ \text{Girls} & 80 & 70 & 65 \end{array}$ (d) $\begin{array}{ccc} & \text{Boys} & \text{Girls} \\ \text{I} & 80 & 80 \\ \text{II} & 70 & 75 \\ \text{III} & 65 & 90 \end{array}$

(ii) If Q represents the matrix of number of units of each type produced by factory B for both boys and girls, then Q is given by

(a) $\begin{array}{ccc} & \text{Boys} & \text{Girls} \\ \text{I} & 85 & 50 \\ \text{II} & 65 & 55 \\ \text{III} & 72 & 80 \end{array}$ (b) $\begin{array}{cccc} & \text{I} & \text{II} & \text{III} \\ \text{Boys} & 50 & 55 & 80 \\ \text{Girls} & 85 & 65 & 72 \end{array}$

(c) $\begin{array}{cccc} & \text{I} & \text{II} & \text{III} \\ \text{Boys} & 80 & 75 & 90 \\ \text{Girls} & 80 & 70 & 65 \end{array}$ (d) $\begin{array}{ccc} & \text{Boys} & \text{Girls} \\ \text{I} & 80 & 80 \\ \text{II} & 70 & 75 \\ \text{III} & 65 & 90 \end{array}$

(iii) The total production of sports clothes of each type for boys is given by the matrix

(a) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [165 & 130 & 137] \end{array}$ (b) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [130 & 165 & 137] \end{array}$

(c) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [165 & 135 & 137] \end{array}$ (d) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [137 & 135 & 165] \end{array}$

(iv) The total production of sports clothes of each type for girls is given by the matrix

(a) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [130 & 130 & 170] \end{array}$ (b) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [170 & 130 & 130] \end{array}$

(c) $\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ [130 & 170 & 130] \end{array}$ (d) none of these

(v) Let R be a 3 × 2 matrix that represents the total production of sports clothes of each type for boys and girls, then R is given by

(a) $\begin{bmatrix} 165 & 135 & 137 \\ 130 & 130 & 170 \end{bmatrix}$ (b) $\begin{bmatrix} 130 & 130 & 170 \\ 165 & 135 & 138 \end{bmatrix}$

(c) $\begin{bmatrix} 165 & 130 \\ 135 & 130 \\ 137 & 170 \end{bmatrix}$ (d) $\begin{bmatrix} 130 & 168 \\ 130 & 135 \\ 170 & 137 \end{bmatrix}$

PART - B

SECTION - III

19. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

20. Find the matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

OR

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x) \cdot F(y) = F(x + y)$.

21. If $A=\begin{bmatrix}7 & 8\\ 1 & 9\end{bmatrix}$ and $B=\begin{bmatrix}7 & 12\\ 5 & 1\end{bmatrix}$, then find the matrix C such that $3A + 5B + 2C$ is a null matrix.

22. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ and $A^{-1}=A'$, then find the value of α.

23. Show that $\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}$.

OR

Prove that $\begin{bmatrix}1 & 1\\ 3 & 4\end{bmatrix}$ is the inverse of $\begin{bmatrix}4 & -1\\ -3 & 1\end{bmatrix}$.

24. If area of triangle is 35 sq. units with vertices $(2, -1)$, $(5, 4)$ and $(k, 4)$, then find the value of k.

OR

Find X and Y, if

(i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

25. Find the area of the triangle whose vertices are $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$ and $C(at_3^2, 2at_3)$.

26. If the matrix $A=\begin{bmatrix}1 & -2 & 3\\ 1 & 2 & 1\\ x & 2 & -3\end{bmatrix}$ is singular, then find x.

27. If $A=\begin{bmatrix}-1 & 2 & 0\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$, then show that $A^{-1} = A^2$.

28. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

SECTION - IV

29. Find x, y and z, if $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies $A' = A^{-1}$.

30. If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$. and $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

31. Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$.

OR

Using matrices, solve the following system of equations : $x + y - z = 3$; $2x + 3y + z = 10$; $3x - y - 7z = 1$.

32. Show that

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\neq\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}.$$

33. If $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$.

34. If $f(x)=\begin{vmatrix}0 & x-a & x-b\\ x+a & 0 & x-c\\ x+b & x+c & 0\end{vmatrix}$, then show that $f(1)\neq 0$.

OR

Find the equation of line joining $P(11, 7)$ and $Q(5, 5)$ using determinant. Also, find the value of k, if $R(-1, k)$ is the point such that area of ΔPQR is 9 sq. units.

35. Express the matrix $A=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

SECTION - V

36. If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}$, find A^{-1}.

Hence solve the system of equations $x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

OR

Find the adjoint of the matrix

$A=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and hence show that $A(\text{adj } A)$ $= |A|\, I_3$.

37. Using matrix method, solve the system of equations $3x + 2y - 2z = 3$, $x + 2y + 3z = 6$ and $2x - y + z = 2$.

OR

If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, then find A^{-1}. Using A^{-1} solve the system of equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

38. If $A=\begin{bmatrix}3 & \sqrt{3} & 2\\ 4 & 2 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$, then verify that (i) $(A')' = A$ (ii) $(A + B)' = A' + B'$

OR

If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying $AA^T = 9I_3$, then find the values of a and b.

SOLUTIONS

1. By comparing the matrices, we get
$x - y = 5$; $2z + w = 3$; $2x - y = 12$ and $2x + w = 15$
Solving these equations, we get
$x = 7, y = 2, w = 1$ and $z = 1$

2. Given that, $A=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$

$\Rightarrow\ A^2=\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$

Also, $B = A^2 \Rightarrow \begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$

Clearly, this is not satisfied by any real value of α.

3. Since, $\begin{bmatrix}a+b & 2\\ 5 & ab\end{bmatrix}=\begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$

$\Rightarrow\ a + b = 6$ and $ab = 8$

$\Rightarrow\ a+\dfrac{8}{a}=6$ $\qquad$ $(\because ab = 8 \Rightarrow b = 8/a)$

$\Rightarrow\ a^2 - 6a + 8 = 0$

$\Rightarrow\ a = 2, 4\ \ \therefore\ a = 2, b = 4$ or $a = 4, b = 2$

OR

Here, $a_{11} = 1 + 1 = 2$, $a_{12} = 1 + 2 = 3$,
$a_{21} = 2 + 1 = 3$ and $a_{22} = 2 + 2 = 4$

Hence, $A=\begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}$

4. We have, $\begin{vmatrix}x & -7\\ x & 5x+1\end{vmatrix}=x(5x+1)+7(x)$

$= 5x^2 + x + 7x = 5x^2 + 8x = x(5x + 8)$

5. We have, $\begin{vmatrix}\cos 15^\circ & \sin 15^\circ\\ \sin 75^\circ & \cos 75^\circ\end{vmatrix}$

$= \cos 75^\circ \cos 15^\circ - \sin 75^\circ \sin 15^\circ = \cos(75^\circ + 15^\circ) = \cos 90^\circ = 0$

6. Here, $a_{11}=\dfrac{(1+2\times 1)^2}{2}=\dfrac{9}{2}$, $a_{12}=\dfrac{(1+2\times 2)^2}{2}=\dfrac{25}{2}$,

$a_{21}=\dfrac{(2+2\times 1)^2}{2}=8$ and $a_{22}=\dfrac{(2+2\times 2)^2}{2}=18$

So, the required matrix $A = \begin{bmatrix}9/2 & 25/2\\ 8 & 18\end{bmatrix}$

7. Given, $|A| = 9$
We know that, $|k \text{ adj } A| = k^n |A|^{n-1}$, where n is the order of matrix A.
$\therefore\ \ |2\cdot\text{adj } A| = 2^3 \cdot 9^2 = 8 \times 81 = 648$.

OR

We have, $A(\text{adj } A) = \begin{bmatrix}8 & 0\\ 0 & 8\end{bmatrix}$

$\Rightarrow\ |A|\cdot I = 8\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\qquad$ $[\because A\,(\text{adj } A) = |A|\cdot I]$

$\Rightarrow\ |A| = 8$

8. By definition of equal matrices, we have

$2x + 1 = x + 3 \Rightarrow x = 2$

$2y = 10 \Rightarrow y = 5$

$\therefore \quad x - y = 2 - 5 = -3$

9. We have, $a_{ij} = \dfrac{(i+2j)^2}{4}$

$$\therefore \quad a_{23} = \frac{[2+2(3)]^2}{4} = \frac{(8)^2}{4} = \frac{64}{4} = 16$$

$$a_{13} = \frac{[1+2(3)]^2}{4} = \frac{(7)^2}{4} = \frac{49}{4}$$

$$\therefore \quad a_{23} \times a_{13} = 16 \times \frac{49}{4} = 196$$

10. By definition of equal matrices, we have $x = 3$

$\therefore \quad x - y = 1 \Rightarrow 3 - y = 1 \Rightarrow y = 2$

OR

We have, $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix}$

$= \sin 10^\circ \cos 80^\circ + \cos 10^\circ \sin 80^\circ = \sin(10^\circ + 80^\circ)$

$= \sin 90^\circ = 1$

11. We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\Rightarrow \quad 5(-2x + 18) - 3(14 + 27) - 1(-42 - 9x) = 0$

$\Rightarrow \quad -x + 9 = 0 \;\Rightarrow\; x = 9$

12. Consider, $(A'A)' = A'((A)')' \qquad [\because \; (AB)' = B'A']$

$\qquad\qquad = A'A \qquad [\because \; (A')' = A]$

Hence $A'A$ is symmetric matrix for any matrix A.

Similarly, $(AA')' = (A')'A' = AA'$

Hence, AA' is also symmetric matrix for any matrix A.

13. Given, $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix}$$

$$\therefore \quad |AB| = \begin{vmatrix} -1 & 5 \\ 4 & 8 \end{vmatrix} = (-1)(8) - 4(5) = -28$$

OR

We have, $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$

Here, $a_{11} = 2$, $a_{22} = 4$, $b_{11} = 2$, $b_{22} = 4$

$\therefore \quad a_{11}b_{11} + a_{22}b_{22} = 2(2) + 4(4) = 4 + 16 = 20$

14. A square matrix A is said to be symmetric if $A' = A$

$$\text{So, } A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

$\Rightarrow \quad A' = A$

So, A is a symmetric matrix.

15. The given points are collinear, iff

$$\begin{vmatrix} 2 & -3 & 1 \\ x & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(-1 - 4) + 3(x - 0) + 1(4x - 0) = 0$

$\Rightarrow \quad -10 + 7x = 0$

$\Rightarrow \quad 7x = 10 \;\Rightarrow\; x = \dfrac{10}{7}$

16. We have, $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$

$\Rightarrow \quad 2x(x + 1) - 2(x + 1)(x + 3) = 3 - 15$

$\Rightarrow \quad 2x^2 + 2x - 2(x^2 + 4x + 3) = -12$

$\Rightarrow \quad 2x^2 + 2x - 2x^2 - 8x - 6 = -12 \Rightarrow -6x = -12 + 6$

$\Rightarrow \quad x = \dfrac{-6}{-6} = 1$

OR

Let M_{ij} and A_{ij} respectively denote the minor and co-factors of determinant $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$M_{11} = -3 \,;\, A_{11} = -3$

$M_{12} = -2 \,;\, A_{12} = -2$

$M_{13} = 2 \,;\, A_{13} = 2$

$M_{21} = -2 \,;\, A_{21} = 2$

$M_{22} = -3 \,;\, A_{22} = -3$

$M_{23} = -2 \,;\, A_{23} = 2$

$M_{31} = 2 \,;\, A_{31} = 2$

$M_{32} = -2 \,;\, A_{32} = 2$

$M_{33} = -3 \,;\, A_{33} = -3$

Now, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 1(2) + 2(-3) + 2(2)$

$= 2 - 6 + 4 = 0$

17. (i)-(c), (ii)-(c), (iii)-(b), (iv)-(c), (v)-(c).

18. (i)-(d), (ii)-(a), (iii)-(c), (iv)-(a), (v)-(c).

19. Let $A = [x \ \ y \ \ z]$

$$\therefore \ \begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1}\begin{bmatrix}x & y & z\end{bmatrix}_{1\times3}=\begin{bmatrix}-4&8&4\\-1&2&1\\-3&6&3\end{bmatrix}_{3\times3}$$

$$\Rightarrow \begin{bmatrix}4x&4y&4z\\x&y&z\\3x&3y&3z\end{bmatrix}=\begin{bmatrix}-4&8&4\\-1&2&1\\-3&6&3\end{bmatrix}$$

On comparing the corresponding elements of two matrices, we get $x = -1$; $y = 2$; $z = 1$

Hence, $A = [-1 \ \ 2 \ \ 1]$

20. Given, $2A - 3B + 5C = O$

$\Rightarrow \ 2A = 3B - 5C$

$$\Rightarrow \ A=\frac{1}{2}[3B-5C] \qquad \text{... (i)}$$

$$\text{Now, } 3B-5C=\begin{bmatrix}-16&6&10\\-26&-2&-18\end{bmatrix}$$

From (i), we get

$$A=\frac{1}{2}\begin{bmatrix}-16&6&10\\-26&-2&-18\end{bmatrix}=\begin{bmatrix}-8&3&5\\-13&-1&-9\end{bmatrix}.$$

OR

We are given that

$$F(x)=\begin{bmatrix}\cos x&-\sin x&0\\\sin x&\cos x&0\\0&0&1\end{bmatrix}$$

$$\therefore \ F(y)=\begin{bmatrix}\cos y&-\sin y&0\\\sin y&\cos y&0\\0&0&1\end{bmatrix}$$

$$F(x)\cdot F(y)=\begin{bmatrix}\cos x&-\sin x&0\\\sin x&\cos x&0\\0&0&1\end{bmatrix}\begin{bmatrix}\cos y&-\sin y&0\\\sin y&\cos y&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y)&-\sin(x+y)&0\\\sin(x+y)&\cos(x+y)&0\\0&0&1\end{bmatrix}=F(x+y)$$

$\therefore \ \ F(x) \times F(y) = F(x + y).$

21. Since, $3A + 5B + 2C = O$

$$\Rightarrow \ 3\begin{bmatrix}7&8\\1&9\end{bmatrix}+5\begin{bmatrix}7&12\\5&1\end{bmatrix}+2C=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\Rightarrow \ 2C=\begin{bmatrix}0&0\\0&0\end{bmatrix}-\begin{bmatrix}56&84\\28&32\end{bmatrix}$$

$$\Rightarrow \ C=\frac{1}{2}\begin{bmatrix}-56&-84\\-28&-32\end{bmatrix}=\begin{bmatrix}-28&-42\\-14&-16\end{bmatrix}$$

22. We have,

$$A=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix} \text{ and } A'=\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$$

Since, $A^{-1} = A'$

$\Rightarrow \ \ AA^{-1} = AA'$

$$\Rightarrow \ I=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix} \quad [\because \ AA^{-1}=I]$$

$$\Rightarrow \ \begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha&0\\0&\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

By equality of matrices, we get $\cos^2\alpha + \sin^2\alpha = 1$, which is true for all real values of α.

23. Let $A=\begin{bmatrix}-7&-9&10\\-12&-15&17\\1&1&-1\end{bmatrix}$ and $B=\begin{bmatrix}2&-1&3\\-5&3&1\\-3&2&3\end{bmatrix}$

$$AB=\begin{bmatrix}-7&-9&10\\-12&-15&17\\1&1&-1\end{bmatrix}\begin{bmatrix}2&-1&3\\-5&3&1\\-3&2&3\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I_3 \ \ \therefore \ B=A^{-1}$$

OR

Let $A^{-1}=\begin{bmatrix}1&1\\3&4\end{bmatrix}$ and $A=\begin{bmatrix}4&-1\\-3&1\end{bmatrix}$

Now, $A^{-1} A = I$

$$\therefore \ \ \text{L.H.S.}=\begin{bmatrix}1&1\\3&4\end{bmatrix}\begin{bmatrix}4&-1\\-3&1\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I.$$

So, $\begin{bmatrix}1&1\\3&4\end{bmatrix}$ is the inverse of $\begin{bmatrix}4&-1\\-3&1\end{bmatrix}$.

24. Area of $\Delta=\dfrac{1}{2}\begin{vmatrix}2&-1&1\\5&4&1\\k&4&1\end{vmatrix}=\pm 35$

$\Rightarrow \quad \frac{1}{2}[25-5k]=\pm 35 \Rightarrow 25-5k=\pm 70$

$\therefore \quad 25 - 5k = 70$ or $25 - 5k = -70$

$\Rightarrow \quad -5k = 45$ or $5k = 95 \Rightarrow k = -9$ or $k = 19$

OR

We are given that $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ...(1)

and $X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ...(2)

Adding (1) and (2), we get

$$\Rightarrow \quad 2X=\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Subtracting (2) from (1), we get

$$\Rightarrow \quad 2Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}-\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}=\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \Rightarrow Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

25. Area of $\Delta ABC=\frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$

$$=\frac{1}{2}\left[2a^2t_1^2t_2-2a^2t_1^2t_3-2a^2t_1t_2^2+2a^2t_2^2t_3+2a^2t_1t_3^2-2a^2t_2t_3^2\right]$$

$$=a^2(t_2-t_3)\left[t_1^2-t_1t_2-t_1t_3+t_2t_3\right]$$

$$= a^2(t_1-t_2)(t_2-t_3)(t_1-t_3)$$

26. Given, matrix A is singular $\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix}=0$$

Applying $R_1 \to R_1 + R_3$, we get

$$\begin{vmatrix} x+1 & 0 & 0 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix}=0$$

Expanding along R_1, we get $(x + 1)(-6 - 2) = 0$

$\Rightarrow -8(x + 1) = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$

27. By definition, a matrix B is an inverse of A if $AB = I = BA$

To prove A^2 is inverse of A. It is sufficient to prove that $A \cdot A^2 = I = A^2A$ *i.e.*, $A^3 = I$.

$$\text{Now, } A^2=A\cdot A=\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^3=A^2\cdot A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

Hence, $A^{-1} = A^2$.

28. $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

L.H.S. $= A^2 - 5A + 7I$

$$=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}-5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}+7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=O$$

Hence proved.

29. Given, $A' = A^{-1}$

$\therefore \quad AA' = AA^{-1}$ [Multiplying by A on both sides]

$\Rightarrow \quad AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad 2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$ ...(i)

and $4y^2 + z^2 = 1$

$\Rightarrow \quad 2 \cdot z^2 + z^2 = 1$ [Using (i)]

$\Rightarrow \quad z=\pm\frac{1}{\sqrt{3}}$ From (i) $y^2=\frac{z^2}{2} \Rightarrow y=\pm\frac{1}{\sqrt{6}}$

Also, $x^2 + y^2 + z^2 = 1$

$$\Rightarrow \quad x^2=1-y^2-z^2=1-\frac{1}{6}-\frac{1}{3}=\frac{1}{2} \quad \Rightarrow \quad x=\pm\frac{1}{\sqrt{2}}$$

$$\therefore \; x=\pm\frac{1}{\sqrt{2}},\, y=\pm\frac{1}{\sqrt{6}} \text{ and } z=\pm\frac{1}{\sqrt{3}}$$

30. We have $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Now, $A^3 - 6A^2 + 9A - 4I = O$

$\Rightarrow \quad 4I = A^3 - 6A^2 + 9A$

Pre multiplying both sides by A^{-1}, we get

$4A^{-1}I = A^{-1}A^3 - 6A^{-1}A^2 + 9A^{-1}A$

$\Rightarrow \quad 4A^{-1} = A^2 - 6A + 9I$

$$\Rightarrow \quad A^{-1} = \frac{1}{4}A^2 - \frac{6}{4}A + \frac{9}{4}I$$

$$= \frac{1}{4}\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix} - \frac{6}{4}\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix} + \frac{9}{4}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix}6-12+9 & -5+6+0 & 5-6+0\\ -5+6+0 & 6-12+9 & -5+6+0\\ 5-6+0 & -5+6+0 & 6-12+9\end{bmatrix} = \frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

31. $A = \begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix} \Rightarrow A' = \begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$

$$A + A' = \begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix} + \begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\Rightarrow \quad \frac{1}{2}(A + A') = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} = 0$$

Now, $A - A' = \begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix} - \begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$

$$= \begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$$

$$\therefore \quad \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix} = \begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix} = A$$

OR

The given system of equations can be written as

$$\begin{bmatrix}1 & 1 & -1\\ 2 & 3 & 1\\ 3 & -1 & -7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ 10\\ 1\end{bmatrix} \text{ i.e., } AX = B,$$

where $A = \begin{bmatrix}1 & 1 & -1\\ 2 & 3 & 1\\ 3 & -1 & -7\end{bmatrix}$, $X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B = \begin{bmatrix}3\\ 10\\ 1\end{bmatrix}$

Now, $|A| = \begin{vmatrix}1 & 1 & -1\\ 2 & 3 & 1\\ 3 & -1 & -7\end{vmatrix}$

$= 1(-21 + 1) - 1(-14 - 3) - 1(-2 - 9)$

$= -20 + 17 + 11 = 8 \neq 0$

$\therefore \quad A^{-1}$ exists.

So, the system of equations has a unique solution which given by $X = A^{-1}B$.

To find A^{-1}: $A_{11} = -20$, $A_{12} = 17$, $A_{13} = -11$, $A_{21} = 8$, $A_{22} = -4$, $A_{23} = 4$, $A_{31} = 4$, $A_{32} = -3$, $A_{33} = 1$

$$\text{Now, adj } A = \begin{bmatrix}-20 & 17 & -11\\ 8 & -4 & 4\\ 4 & -3 & 1\end{bmatrix}' = \begin{bmatrix}-20 & 8 & 4\\ 17 & -4 & -3\\ -11 & 4 & 1\end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{8}\begin{bmatrix}-20 & 8 & 4\\ 17 & -4 & -3\\ -11 & 4 & 1\end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{8}\begin{bmatrix}-20 & 8 & 4\\ 17 & -4 & -3\\ -11 & 4 & 1\end{bmatrix}\begin{bmatrix}3\\ 10\\ 1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{8}\begin{bmatrix}24\\ 8\\ 8\end{bmatrix} = \begin{bmatrix}3\\ 1\\ 1\end{bmatrix} \Rightarrow x = 3, y = 1, z = 1$$

32. Let $A = \begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$ and $B = \begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}$

$$AB = \begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix} = \begin{bmatrix}5 & 8 & 14\\ 0 & -1 & 1\\ -1 & 0 & 1\end{bmatrix}$$

$$BA = \begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix} = \begin{bmatrix}-1 & -1 & -3\\ 1 & 0 & 0\\ 6 & 11 & 6\end{bmatrix}$$

Hence, L.H.S. ≠ R.H.S.

33. We know that

$$\cos\alpha = \frac{1 - \tan^2\left(\frac{\alpha}{2}\right)}{1 + \tan^2\left(\frac{\alpha}{2}\right)} = \frac{1 - t^2}{1 + t^2}, \text{ where } \tan\frac{\alpha}{2} = t$$

and $\sin\alpha = \dfrac{2\tan\left(\dfrac{\alpha}{2}\right)}{1+\tan^2\left(\dfrac{\alpha}{2}\right)} = \dfrac{2t}{1+t^2}$

Now, $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$

and $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$

$$\therefore\ (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \dfrac{(1-t^2)}{(1+t^2)} & \dfrac{-2t}{(1+t^2)} \\ \dfrac{2t}{(1+t^2)} & \dfrac{(1-t^2)}{(1+t^2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I+A)$$

Hence, $(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = (I+A).$

34. We have, $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$

Put $x = a$, then $f(a) = \begin{vmatrix} 0 & 0 & a-b \\ 2a & 0 & a-c \\ a+b & a+c & 0 \end{vmatrix}$

Expanding along R_1, we get $f(a) = (a - b)\,[2a(a + c)] \neq 0$

Also, $f(b) = \begin{vmatrix} 0 & b-a & 0 \\ b+a & 0 & b-c \\ 2b & b+c & 0 \end{vmatrix}$

Expanding along R_1, we get $f(b) = -(b - a)\,[-2b(b - c)]$ $= 2b(b - a)(b - c) \neq 0$

Also, $f(0) = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$

Expanding along R_1, we get
$f(0) = a(bc) - b(ac) = abc - abc = 0$

And, $f(1) = \begin{vmatrix} 0 & 1-a & 1-b \\ 1+a & 0 & 1-c \\ 1+b & 1+c & 0 \end{vmatrix}$

Expanding along R_1, we get
$f(1) = -(1 - a)\,[0 - (1 - c)(1 + b)] + (1 - b)\,[(1 + a)(1 + c) - 0]$
$= (1 - a)(1 + b)(1 - c) + (1 - b)(1 + a)(1 + c) \neq 0$

Now, pre-multiplying by A^{-1} both sides, we get

$(A^{-1}A)A - 5A^{-1}A + 7A^{-1}I = A^{-1}O$

$\Rightarrow\ IA - 5I + 7A^{-1} = O$

$\Rightarrow\ A - 5I + 7A^{-1} = O \quad \Rightarrow\quad 7A^{-1} = 5I - A$

$\Rightarrow\ 7A^{-1} = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow\ 7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow A^{-1} = \dfrac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

OR

Let $A(x, y)$ be any point on line PQ. Then, the points P, Q and A are collinear.

So, $\begin{vmatrix} 11 & 7 & 1 \\ 5 & 5 & 1 \\ x & y & 1 \end{vmatrix} = 0 \ \Rightarrow\ x - 3y + 10 = 0,$

which is the required equation of line joining points P and Q.

Now, according to the question,

Area of ΔPQR = 9 sq. units.

$\therefore\ \dfrac{1}{2}\begin{vmatrix} 11 & 7 & 1 \\ 5 & 5 & 1 \\ -1 & k & 1 \end{vmatrix} = \pm 9$

$\Rightarrow\ -6k + 18 = \pm 18 \ \Rightarrow\ k = \dfrac{-18 \pm 18}{-6}$

$\therefore$ Required values of k are 0 and 6.

35. We have, $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$

Let $P = \dfrac{1}{2}(A + A') = \begin{bmatrix} 2 & \dfrac{11}{2} & \dfrac{-5}{2} \\ \dfrac{11}{2} & 3 & \dfrac{3}{2} \\ \dfrac{-5}{2} & \dfrac{3}{2} & 4 \end{bmatrix}$

which is a symmetric matrix.

Also, $Q=\frac{1}{2}(A-A')=\begin{bmatrix}0 & \frac{-3}{2} & \frac{-7}{2}\\ \frac{3}{2} & 0 & \frac{7}{2}\\ \frac{7}{2} & \frac{-7}{2} & 0\end{bmatrix}$,

which is a skew-symmetric matrix.

Now, $A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')=P+Q$

$$=\begin{bmatrix}2 & \frac{11}{2} & \frac{-5}{2}\\ \frac{11}{2} & 3 & \frac{3}{2}\\ \frac{-5}{2} & \frac{3}{2} & 4\end{bmatrix}+\begin{bmatrix}0 & \frac{-3}{2} & \frac{-7}{2}\\ \frac{3}{2} & 0 & \frac{7}{2}\\ \frac{7}{2} & \frac{-7}{2} & 0\end{bmatrix}$$

Hence, A is represented as sum of symmetric and skew symmetric matrix.

36. We have, $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}$

Now, $|A| = 1(0-2) - 1(1-6) + 1(1)$
$= -2 + 5 + 1 = 4 \neq 0 \quad \therefore A^{-1}$ exists.

So, $\text{adj } A=\begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}$

$$\therefore \quad A^{-1}=\frac{1}{|A|}\text{adj } A=\frac{1}{4}\begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}$$

The given system of equations can be written as

$AX = B$ *i.e.*, $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 7\\ 12\end{bmatrix}$

$\because \quad A^{-1}$ exists.

So, the system has a unique solution given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}\begin{bmatrix}6\\ 7\\ 12\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12\\ 4\\ 8\end{bmatrix}=\begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$$

$\therefore \quad x = 3, y = 1, z = 2.$

OR

We have, $A=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix} \Rightarrow |A|=\begin{vmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{vmatrix}$

$= (-1)(1-4) - (-2)(2+4) - 2(-4-2) = 3 + 12 + 12$
$= 27 \neq 0$

$\therefore \quad A^{-1}$ exists

$$\therefore \quad \text{adj } A=\begin{bmatrix}-3 & -6 & -6\\ 6 & 3 & -6\\ 6 & -6 & 3\end{bmatrix}'=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$$

$$\therefore \quad A(\text{adj } A)=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$$

$$=\begin{bmatrix}27 & 0 & 0\\ 0 & 27 & 0\\ 0 & 0 & 27\end{bmatrix}=27\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=|A|I_3$$

37. Given system of equations are

$3x + 2y - 2z = 3 \Rightarrow x + 2y + 3z = 6$ and $2x - y + z = 2$

This can be written as

$\begin{bmatrix}3 & 2 & -2\\ 1 & 2 & 3\\ 2 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 6\\ 2\end{bmatrix}$ *i.e.*, $AX = B$, where

$$A=\begin{bmatrix}3 & 2 & -2\\ 1 & 2 & 3\\ 2 & -1 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\ B=\begin{bmatrix}3\\ 6\\ 2\end{bmatrix}$$

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$|A| = 3(2 + 3) - 2(1 - 6) + (-2)(-1 - 4) = 15 + 10 + 10$
$= 35 \neq 0$

$\therefore \quad A^{-1}$ exists

$$\text{Hence, adj } A = \begin{bmatrix} 5 & 5 & -5 \\ 0 & 7 & 7 \\ 10 & -11 & 4 \end{bmatrix}' = \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 15+20 \\ 15+42-22 \\ -15+42+8 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 1$, $y = 1$ and $z = 1$.

OR

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$\Rightarrow \quad |A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$
$= 0 - 6 + 5 = -1 \neq 0$

$\therefore \quad A^{-1}$ exists.

Here, $A_{11} = 0, A_{12} = 2, A_{13} = 1, A_{21} = -1,$
$A_{22} = -9, A_{23} = -5, A_{31} = 2, A_{32} = 23, A_{33} = 13$

$$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Solution of the system of equations is given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = 3$.

38. We have, $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix} \text{ and } B' = \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\text{(i)} \quad (A')' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix}' = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix} = A$$

$$\text{(ii)} \quad A + B = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\Rightarrow (A+B)' = \begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}' = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\text{Also, } A' + B' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

Thus, $(A + B)' = A' + B'$

OR

We have,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

Also, $AA^T = 9I_3$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

On equating the corresponding elements, we get

$a + 2b + 4 = 0$; $2a + 2 - 2b = 0$ and $a^2 + 4 + b^2 = 9$

$\Rightarrow \quad a + 2b + 4 = 0$ ... (i)

$a + 1 - b = 0$ ... (ii)

and $a^2 + b^2 = 5$ ... (iii)

Solving (i) and (ii), we get

$a = -2, b = -1$

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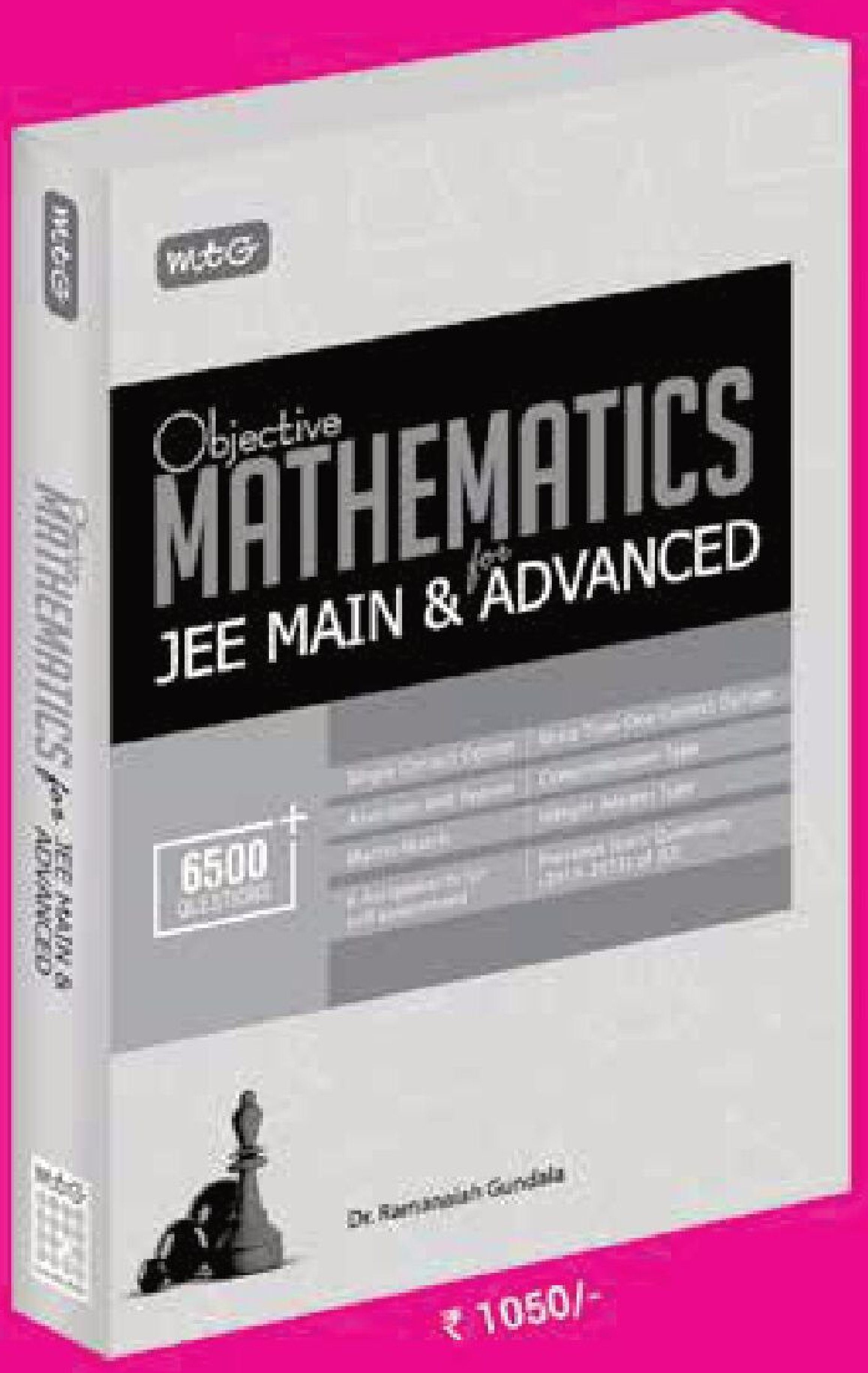
ABRACADABRA



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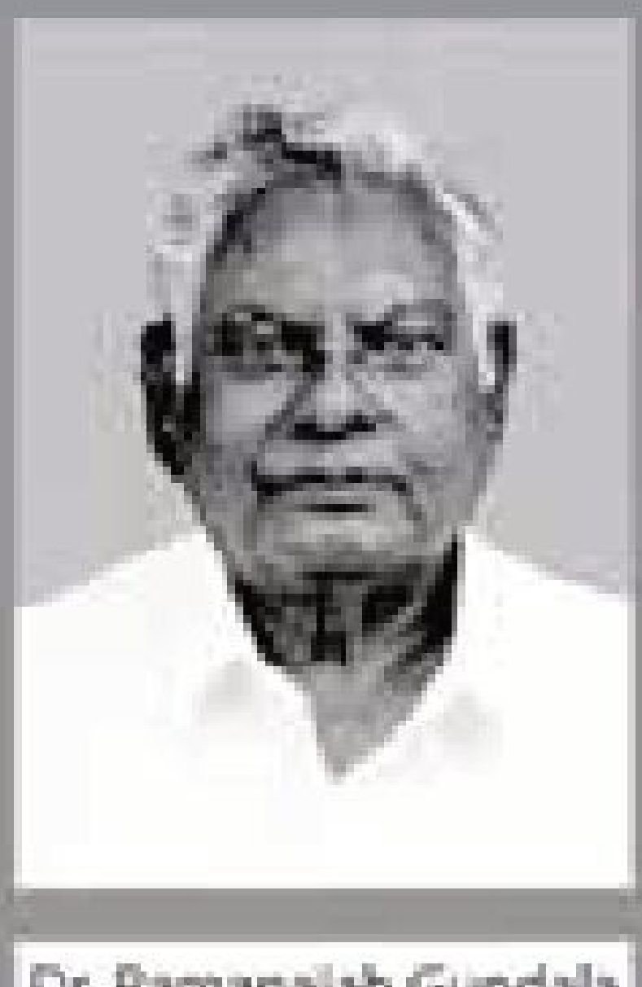

Dr. Ramanaiah Gundala

After securing his DIIT and Ph.D from IIT Kharagpur, Dr Gundala was elected Fellow of National Academy of Sciences (FNASc). His 50+ years of teaching experience includes distinguished tenures at IIT Kharagpur and Anna University, Chennai. He has authored 7 books and published an astonishing 85 research papers. He's now retired and prepares students for success in IIT-JEE at two leading coaching institutes in Chennai and Warangal.

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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 2 : Inverse Trigonometric Functions

Time Taken : 60 Min.

Only One Option Correct Type

1. If $x^2 + y^2 + z^2 = r^2$, then

$$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{zx}{yr}\right) \text{ is equal to}$$

(a) π (b) $\frac{\pi}{2}$

(c) 0 (d) none of these

2. Let $\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = \pi$. If x satisfies the cubic $ax^3 + bx^2 + cx - 1 = 0$, then $(a + b + c)$ is equal to

(a) 24 (b) 25 (c) 26 (d) 27

3. The value of

$$\sec\left[\sin^{-1}\left(-\sin\frac{50\pi}{9}\right) + \cos^{-1}\cos\left(-\frac{31\pi}{9}\right)\right]$$

is equal to

(a) $\sec\frac{10\pi}{9}$ (b) $\sec\frac{\pi}{9}$

(c) 1 (d) -1

4. If $\frac{1}{2}\sin^{-1}\left[\frac{3\sin 2\theta}{5+4\cos 2\theta}\right] = \tan^{-1} x$, then $x =$

(a) $\tan 3\theta$ (b) $3 \tan \theta$

(c) $\left(\frac{1}{3}\right)\tan\theta$ (d) $3 \cot \theta$

5. If $f(x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2} \le x \le 1$,

then $f(x)$ is equal to

(a) $\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(x)$ (b) $\sin^{-1}x - \frac{\pi}{6}$

(c) $\sin^{-1}x + \frac{\pi}{2}$ (d) none of these

6. If $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x] = 1$, where $[\cdot]$ denotes the greatest integer function, then x belongs to the interval :

(a) [tan sin cos 1, tan sin cos sin 1]

(b) (tan sin cos 1, tan sin cos sin 1)

(c) $[-1, 1]$

(d) [sin cos tan 1, sin cos sin tan 1]

One or More Than One Option(s) Correct Type

7. α, β and γ are angles given by $\alpha = 2\tan^{-1}(\sqrt{2}-1)$,

$\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ and

$\gamma = \cos^{-1}\left(\frac{1}{3}\right)$, then

(a) $\alpha > \beta$ (b) $\beta > \gamma$

(c) $\gamma > \alpha$ (d) none of these

8. $2\tan^{-1}(-3)$ is equal to

(a) $-\cos^{-1}\left(-\frac{4}{5}\right)$ (b) $-\pi + \cos^{-1}\left(\frac{4}{5}\right)$

(c) $-\frac{\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right)$ (d) $\cot^{-1}\left(\frac{4}{3}\right)$

9. If $\sin^{-1}\left(a - \frac{a^2}{3} + \frac{a^3}{9} + \ldots\right) + \cos^{-1}(1 + b + b^2 + \ldots)$

$= \frac{\pi}{2}$, then

(a) $b = \frac{2a-3}{3a}$ (b) $b = \frac{3a-2}{2a}$

(c) $a = \frac{3}{2-3b}$ (d) $a = \frac{2}{3-2b}$

10 The value of $\tan^{-1}\sqrt{\dfrac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\dfrac{b(a+b+c)}{ca}}$ $+\tan^{-1}\sqrt{\dfrac{c(a+b+c)}{ab}}$ is, (where $a+b+c\neq 0$)

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) π (d) 0

11. If $\cot^{-1}\left(\dfrac{n^2-10n+21.6}{\pi}\right)>\dfrac{\pi}{6}$, $n\in N$, then n can be

(a) 3 (b) 2 (c) 4 (d) 8

12. $\displaystyle\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

(a) $\tan^{-1}\left(\dfrac{n^2+n}{n^2+n+2}\right)$ (b) $\tan^{-1}\left(\dfrac{n^2-n}{n^2-n+2}\right)$

(c) $\tan^{-1}\left(\dfrac{n^2+n+2}{n^2+n}\right)$ (d) None of these

13. $\theta=\tan^{-1}(2\tan^2\theta)-\tan^{-1}\left\{\left(\dfrac{1}{3}\right)\tan\theta\right\}$, if

(a) $\tan\theta=-2$ (b) $\tan\theta=0$
(c) $\tan\theta=1$ (d) $\tan\theta=2$

Comprehension Type

Let $a=\cos^{-1}\cos 20$, $b=\cos^{-1}\cos 30$ and $c=\sin^{-1}\sin(a+b)$, then

14. The largest integer x for which $\sin^{-1}(\sin x)\geq |x-(a+b+c)|$ is

(a) 1 (b) 2 (c) 3 d) 4

15. If $5\sec^{-1}x+10\sin^{-1}y=10(a+b+c)$, then the value of $\tan^{-1}x+\cos^{-1}(y-1)$ is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$

(c) π (d) 0

Matrix Match Type

16. Match the following:

Column-I		Column-II	
P.	If $A=\tan^{-1}\dfrac{x\sqrt{3}}{2\lambda-x}$ and $B=\tan^{-1}\left(\dfrac{2x-\lambda}{\lambda\sqrt{3}}\right)$, then the value of $A-B$ is	1.	$\dfrac{\pi}{5}$
Q.	If $\tan^{-1}x+\tan^{-y}=\dfrac{4\pi}{5}$ then $\cot^{-1}x+\cot^{-1}y$ equal to	2.	$\dfrac{\pi}{4}$
R.	$\tan^{-1}\dfrac{1}{7}+2\tan^{-1}\dfrac{1}{3}=$	3.	$\dfrac{\pi}{2}$
S.	$\sin^{-1}\left(\dfrac{4}{5}\right)+2\tan^{-1}\left(\dfrac{1}{3}\right)=$	4.	$\dfrac{\pi}{6}$

	P	Q	R	S
(a)	1	3	4	2
(b)	2	4	3	1
(c)	4	2	3	1
(d)	4	1	2	3

Numerical Answer Type

17. The value of

$$\frac{1}{\pi}\left\{216\sin^{-1}\left(\sin\frac{7\pi}{6}\right)+27\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+28\tan^{-1}\left(\tan\frac{5\pi}{4}\right)+200\cot^{-1}\left(\cot\left(\frac{-\pi}{4}\right)\right)\right\}$$

must be equal to __________ .

18. If $\sin^{-1}x+\sin^{-1}y=\pi$ and if $x=\lambda y$, then the value of $39^{2\lambda}+5^{\lambda}$ must be equal to __________ .

19. If $x=\tan\left[\cos^{-1}\dfrac{1}{5\sqrt{2}}-\sin^{-1}\dfrac{4}{\sqrt{17}}\right]$, then $29x$ is equal to

20. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{3\pi}{2}$, then the value of $x^{100}+y^{100}+z^{100}-\dfrac{9}{x^{101}+y^{101}+z^{101}}$ is equal to __________ .

◆ ◆

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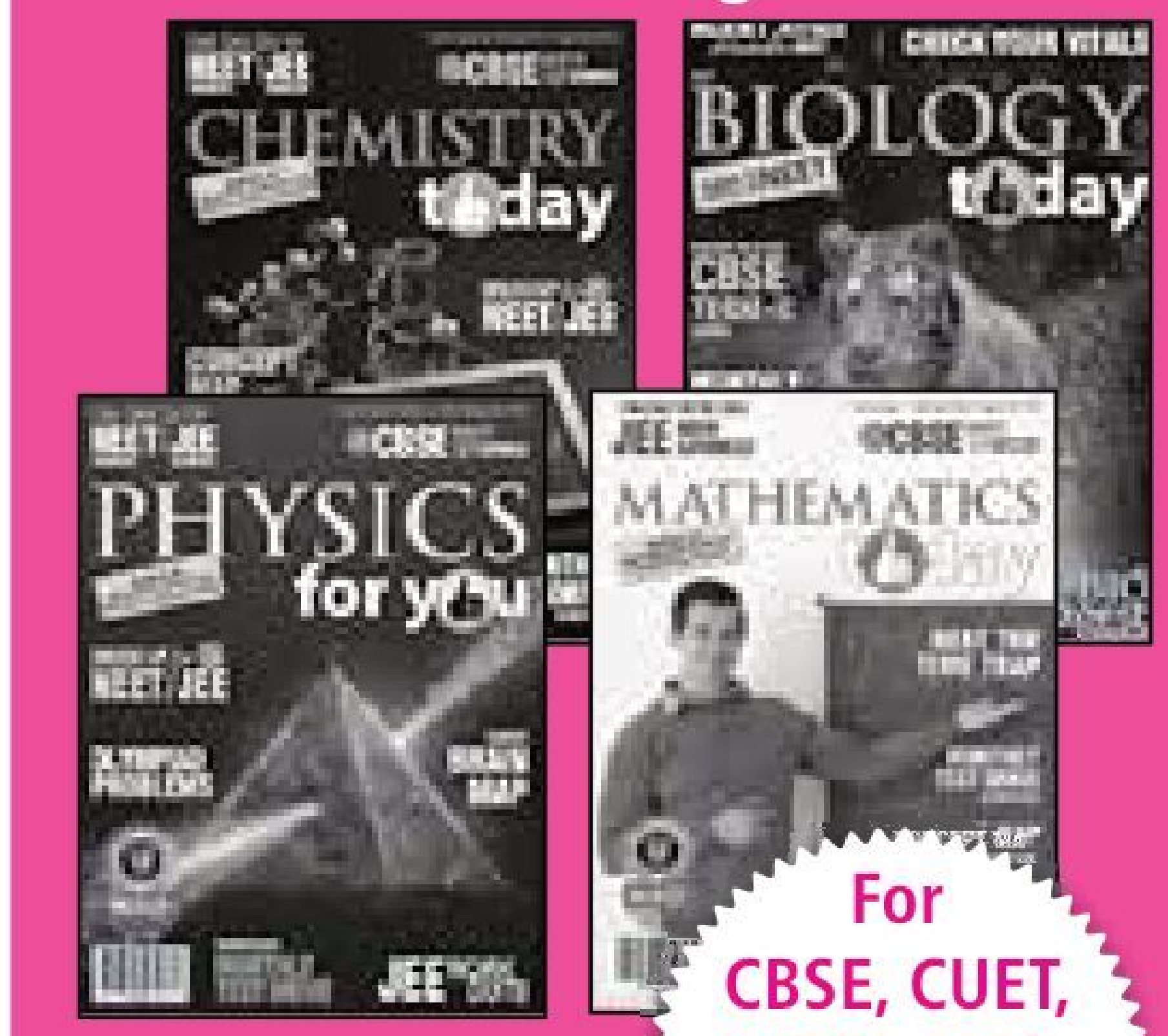

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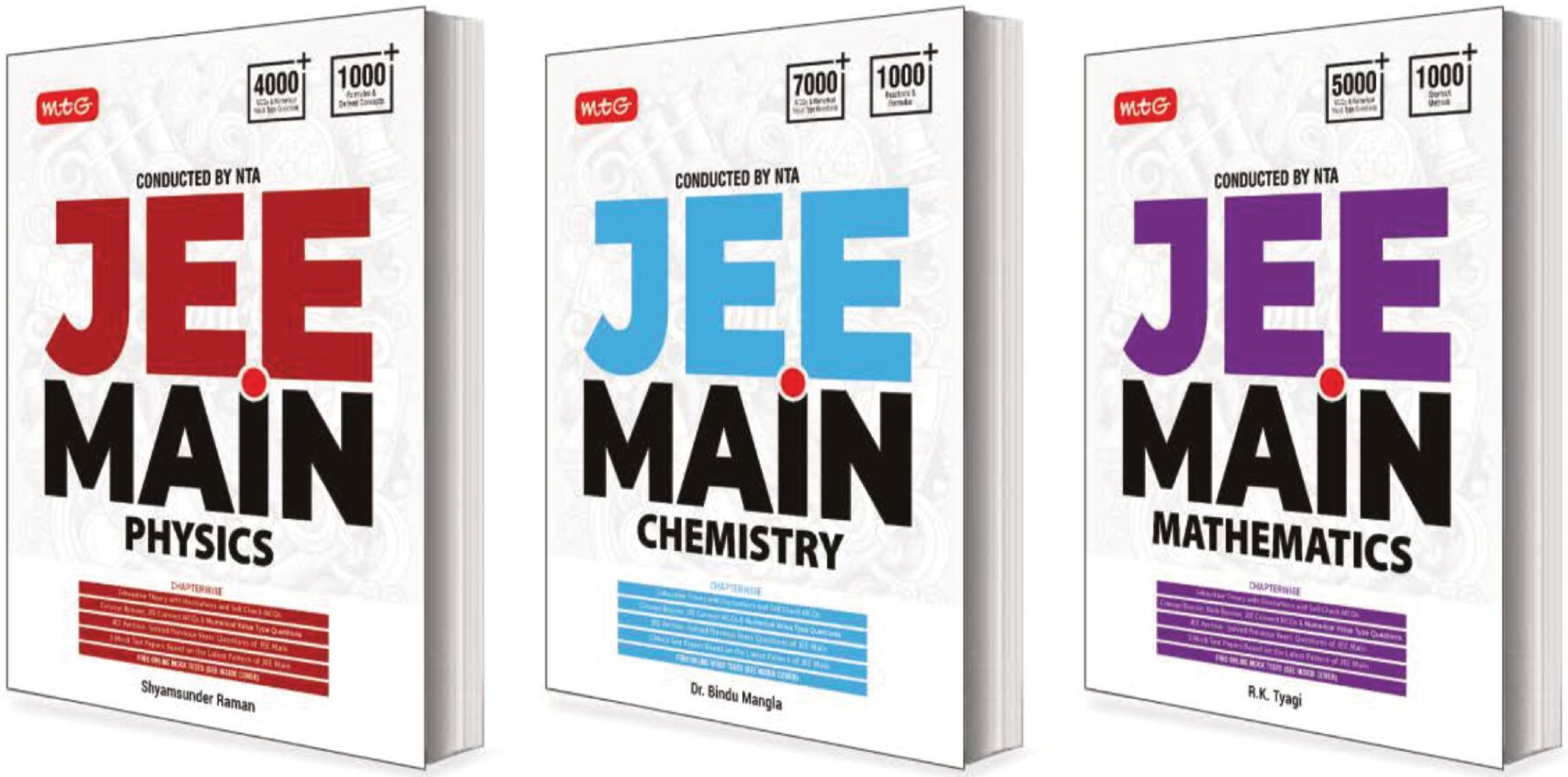

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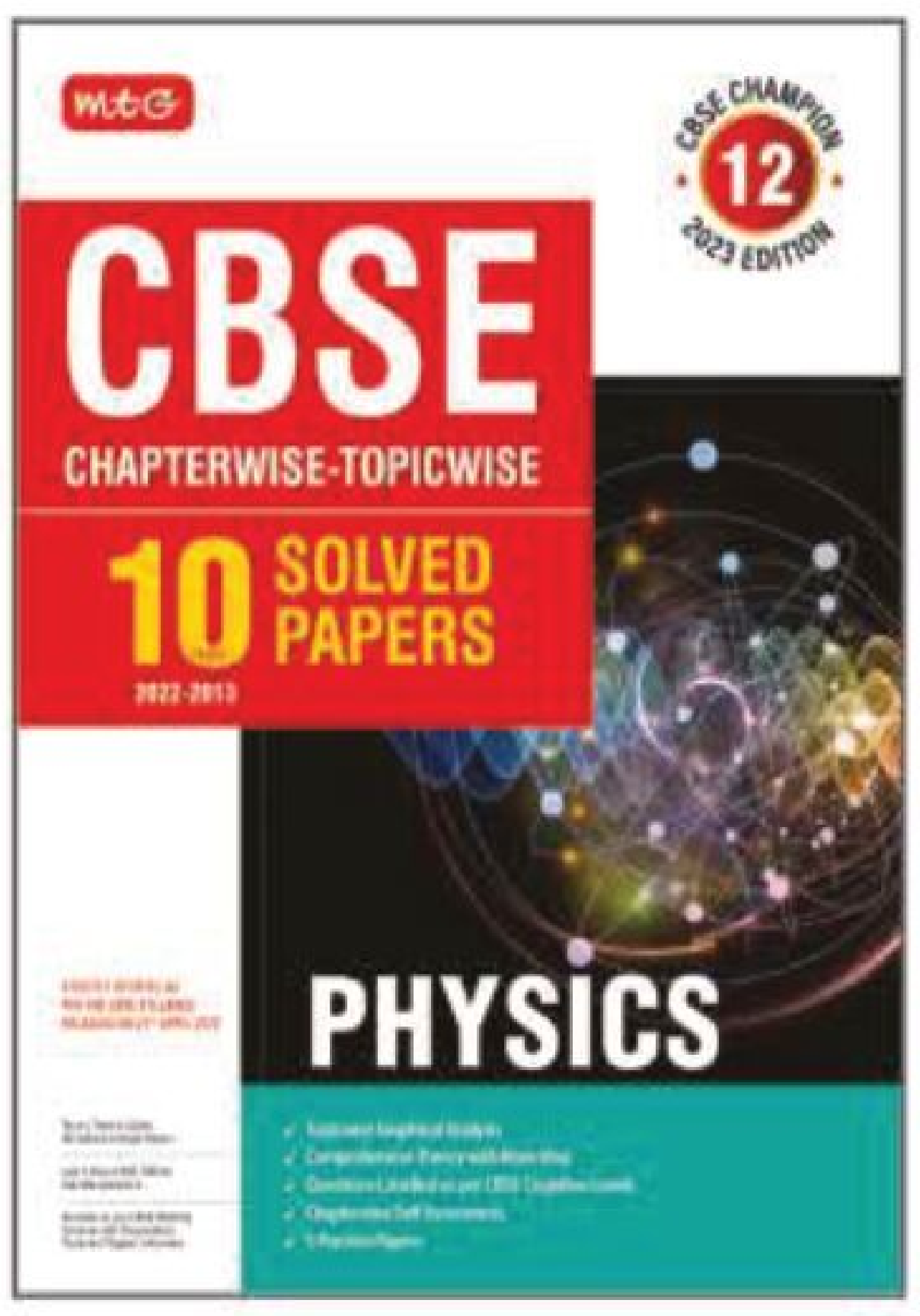
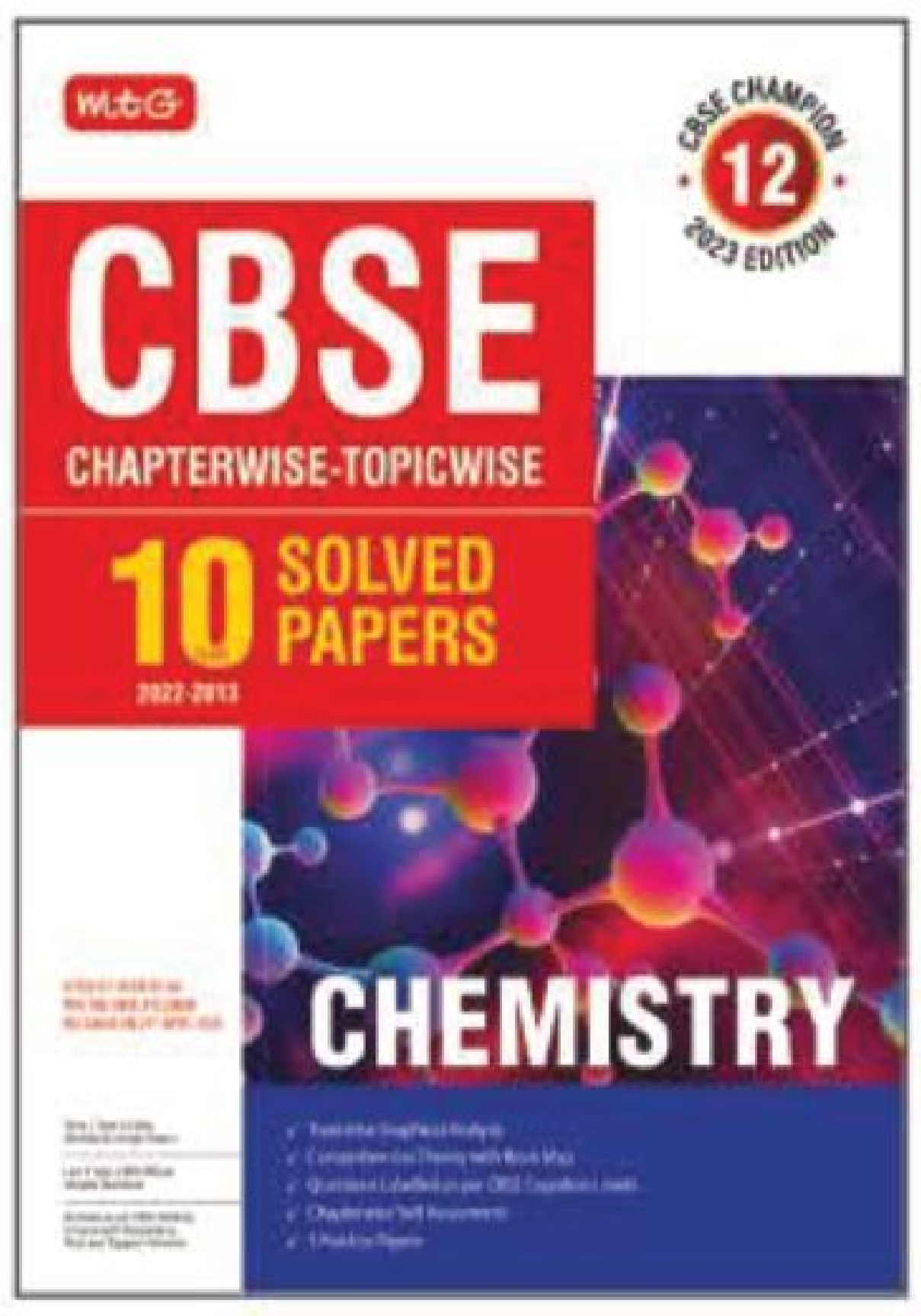
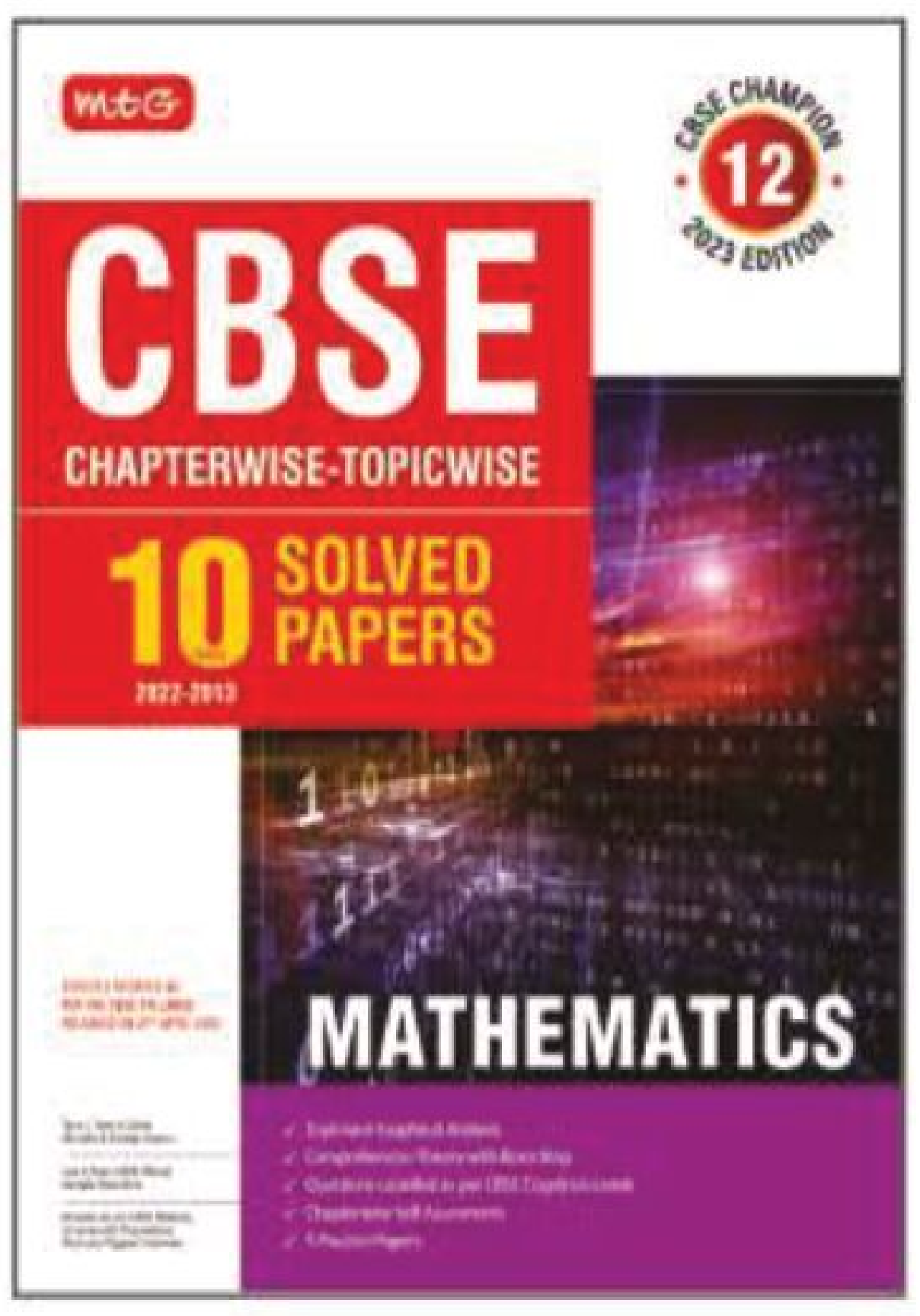
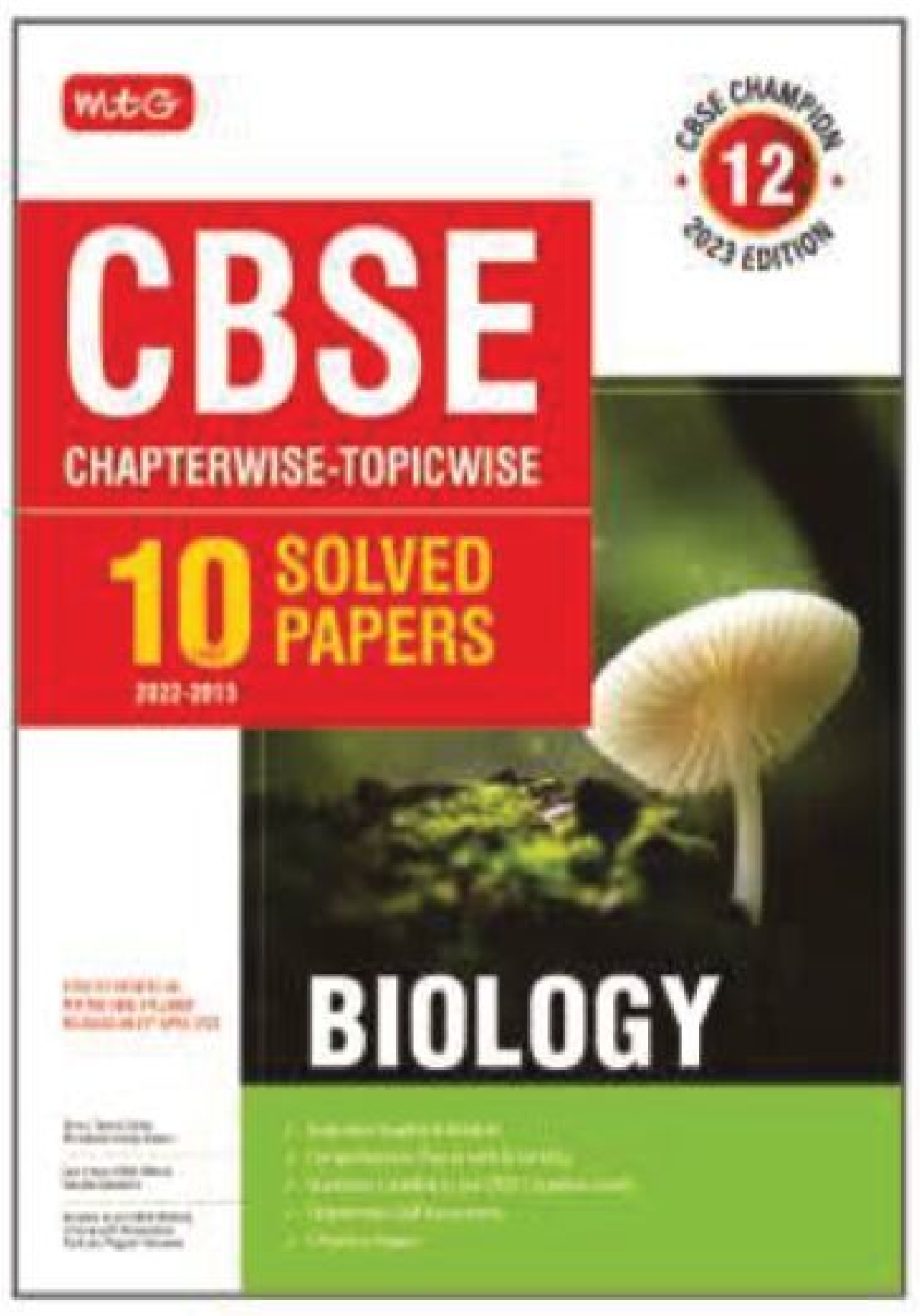
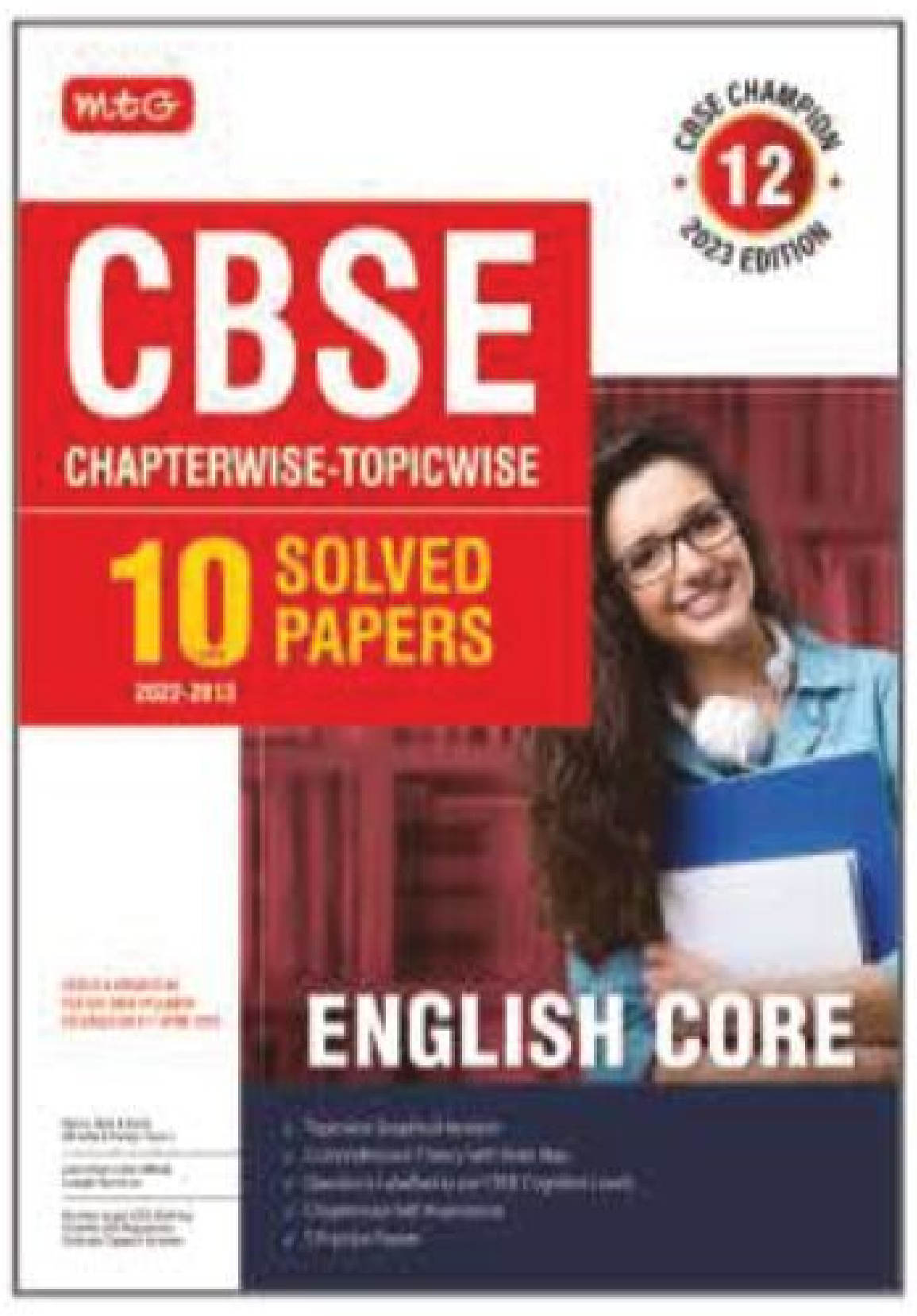
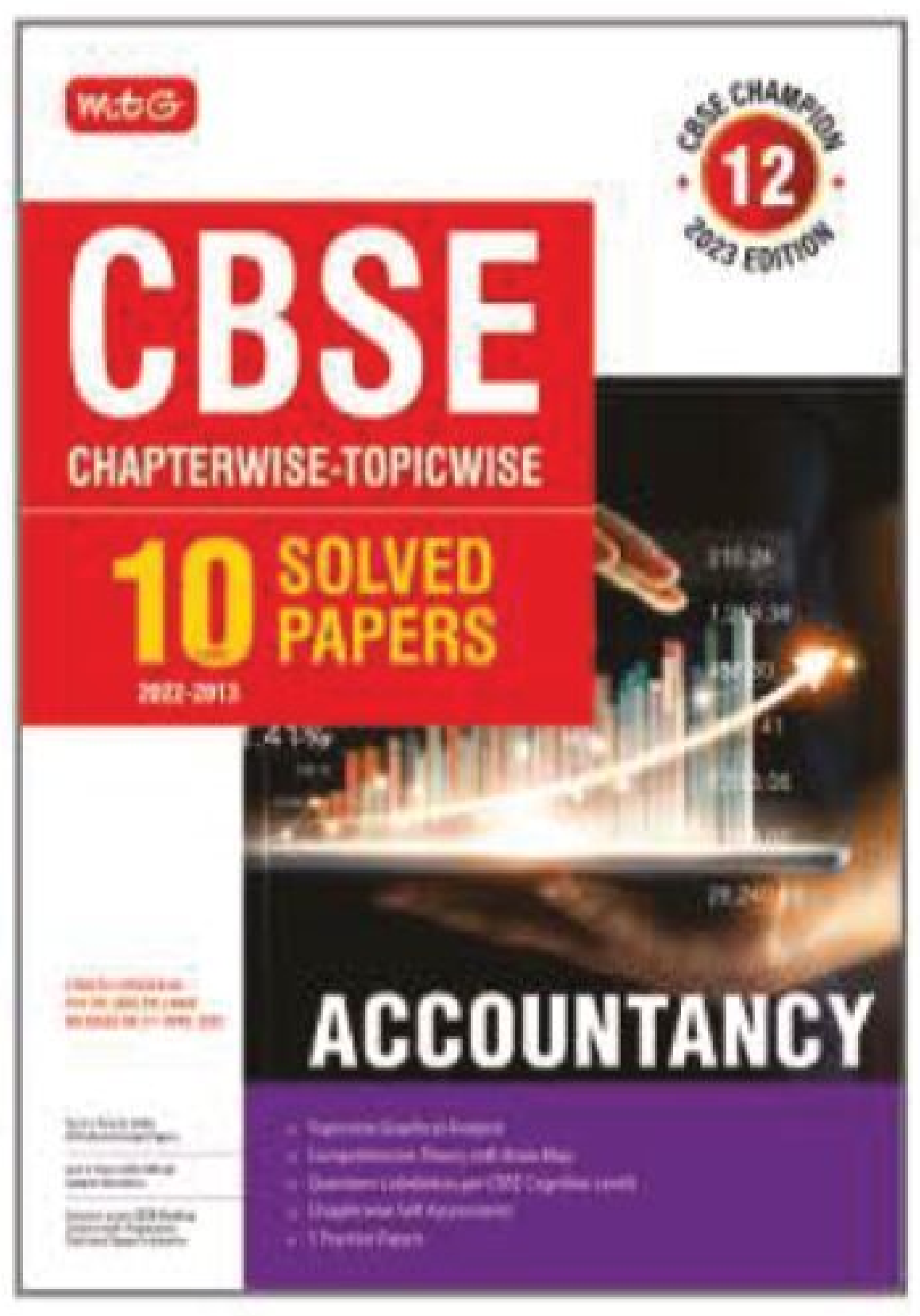
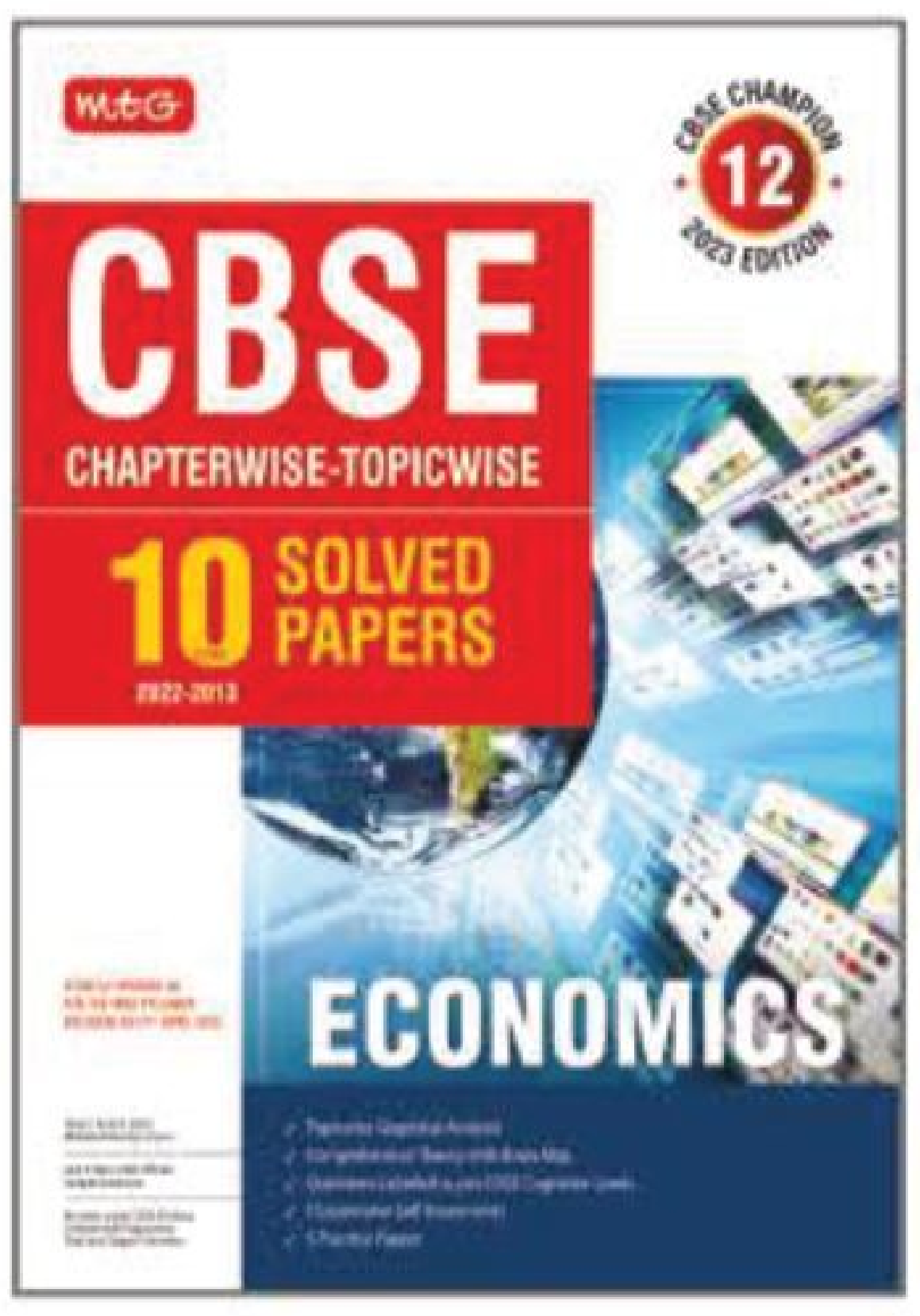
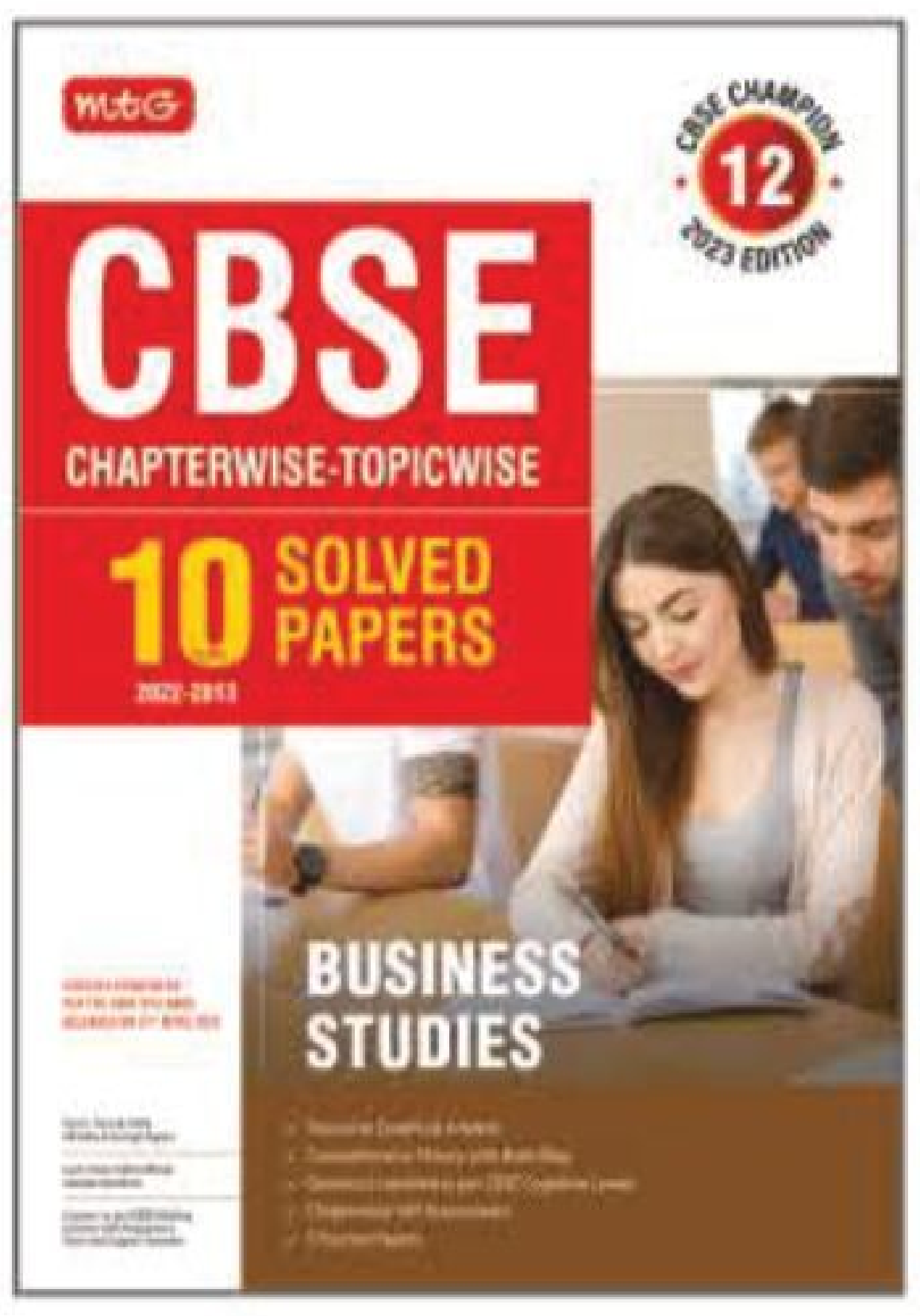
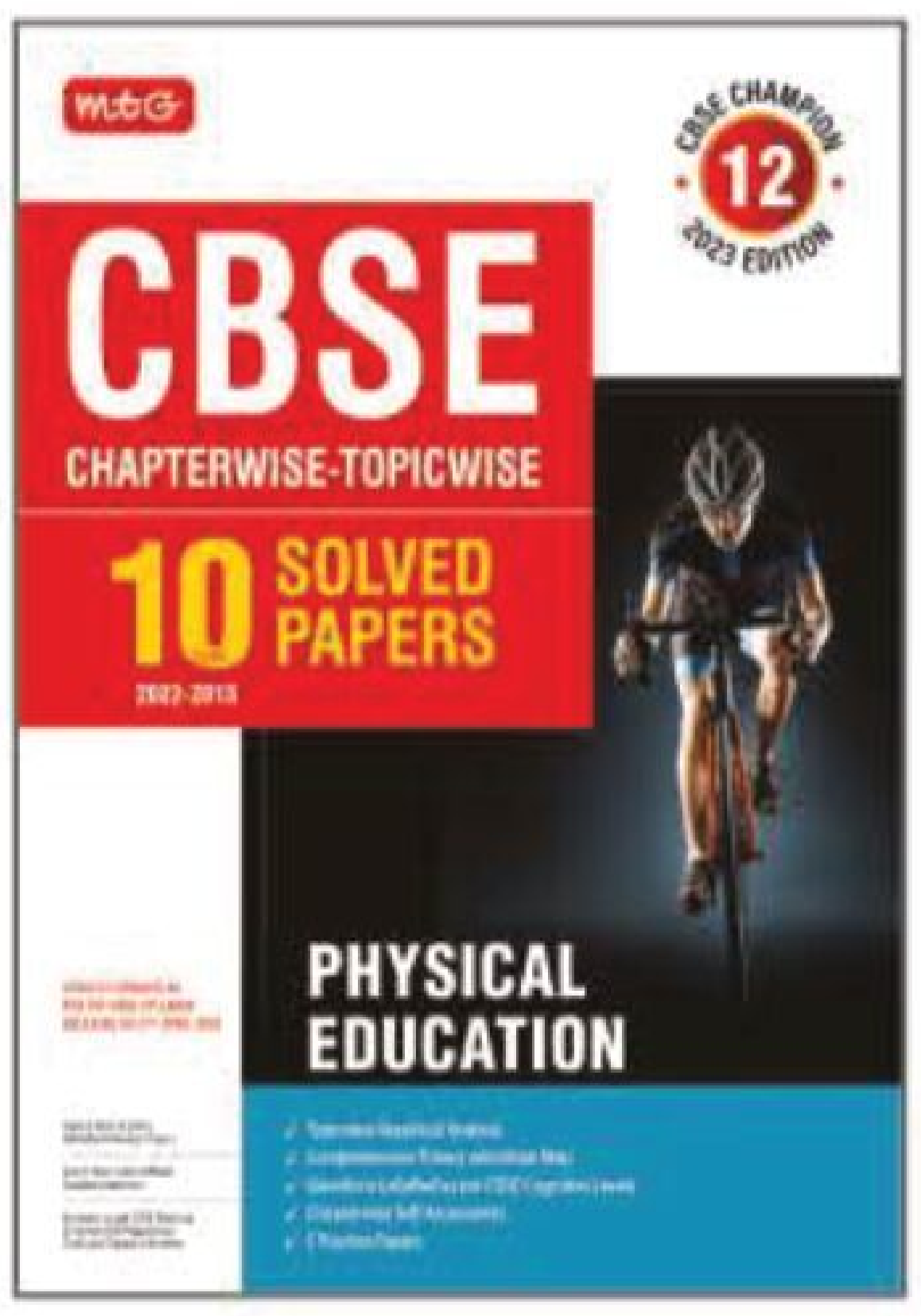
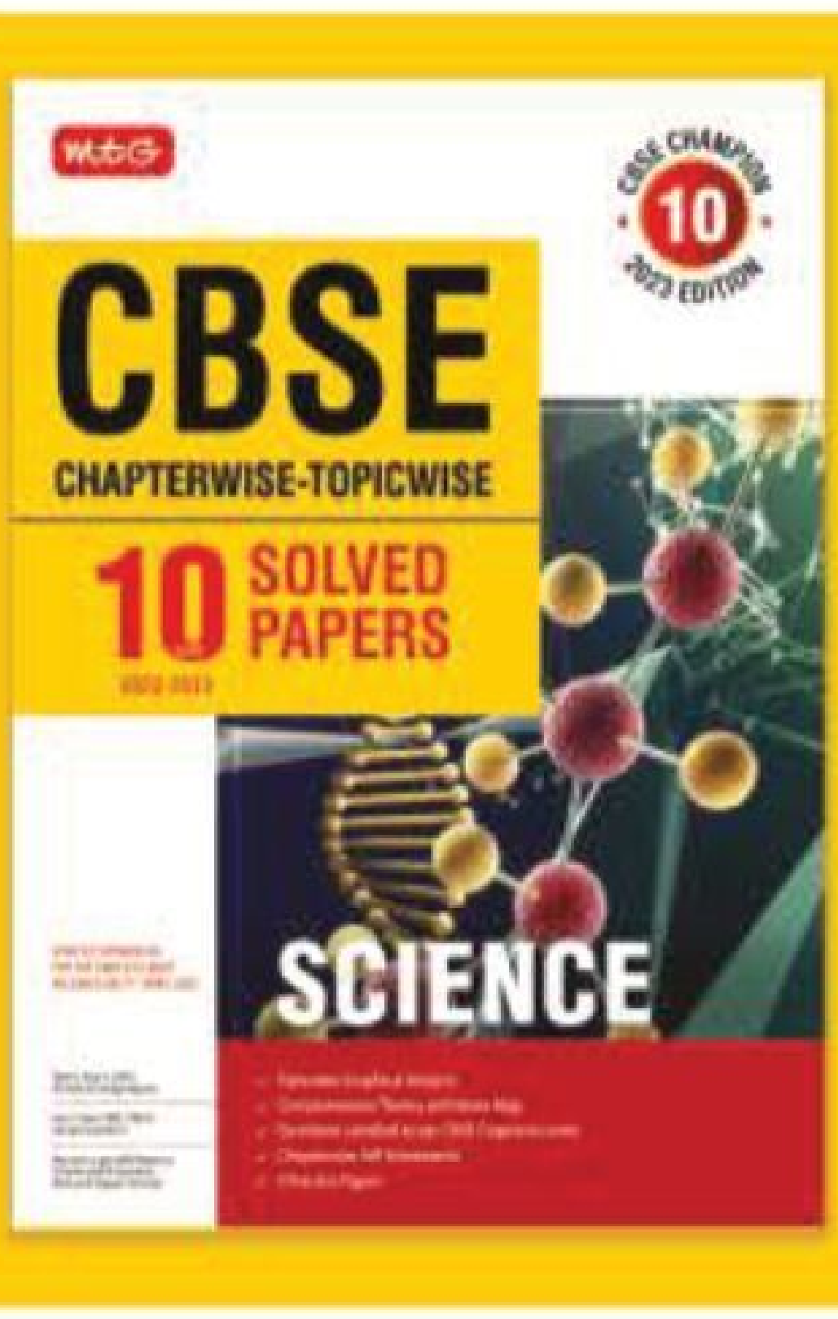
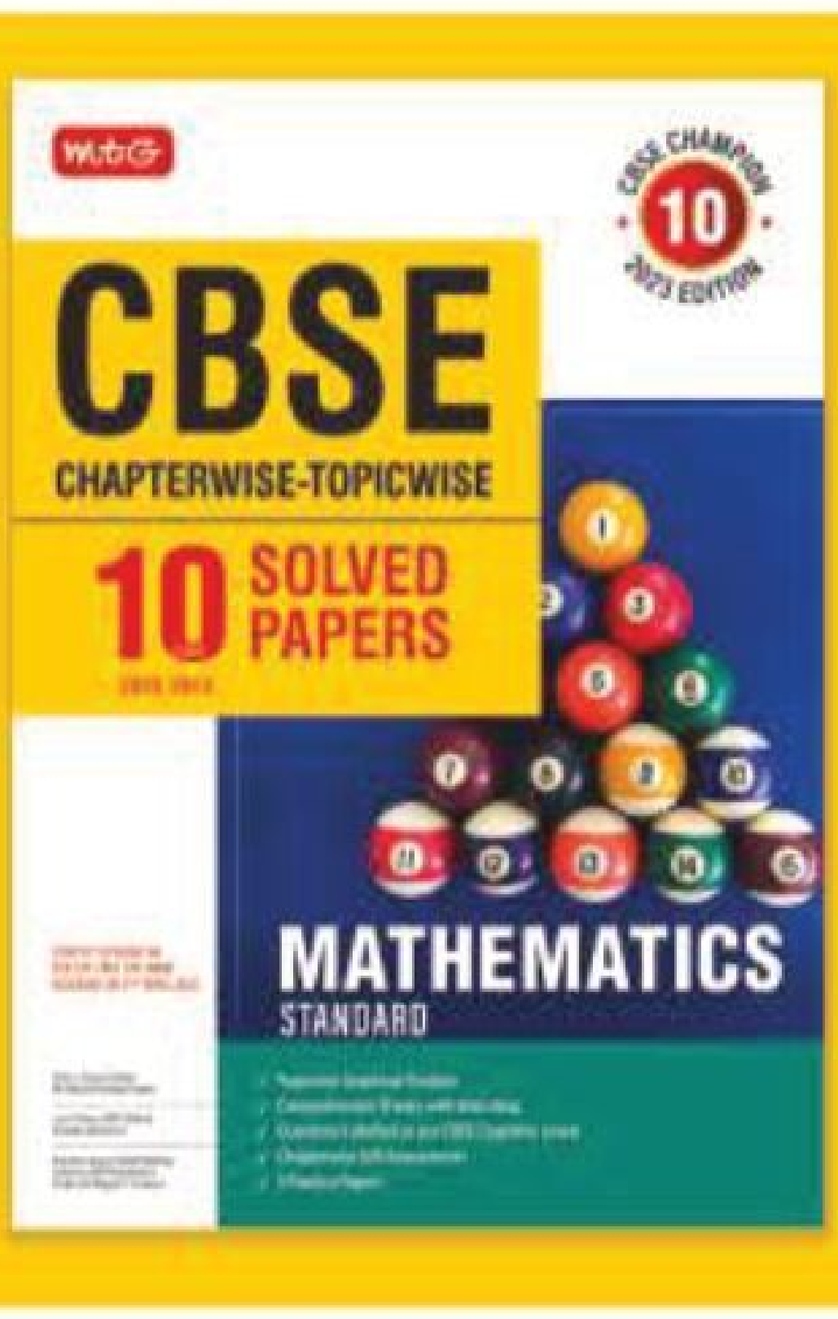
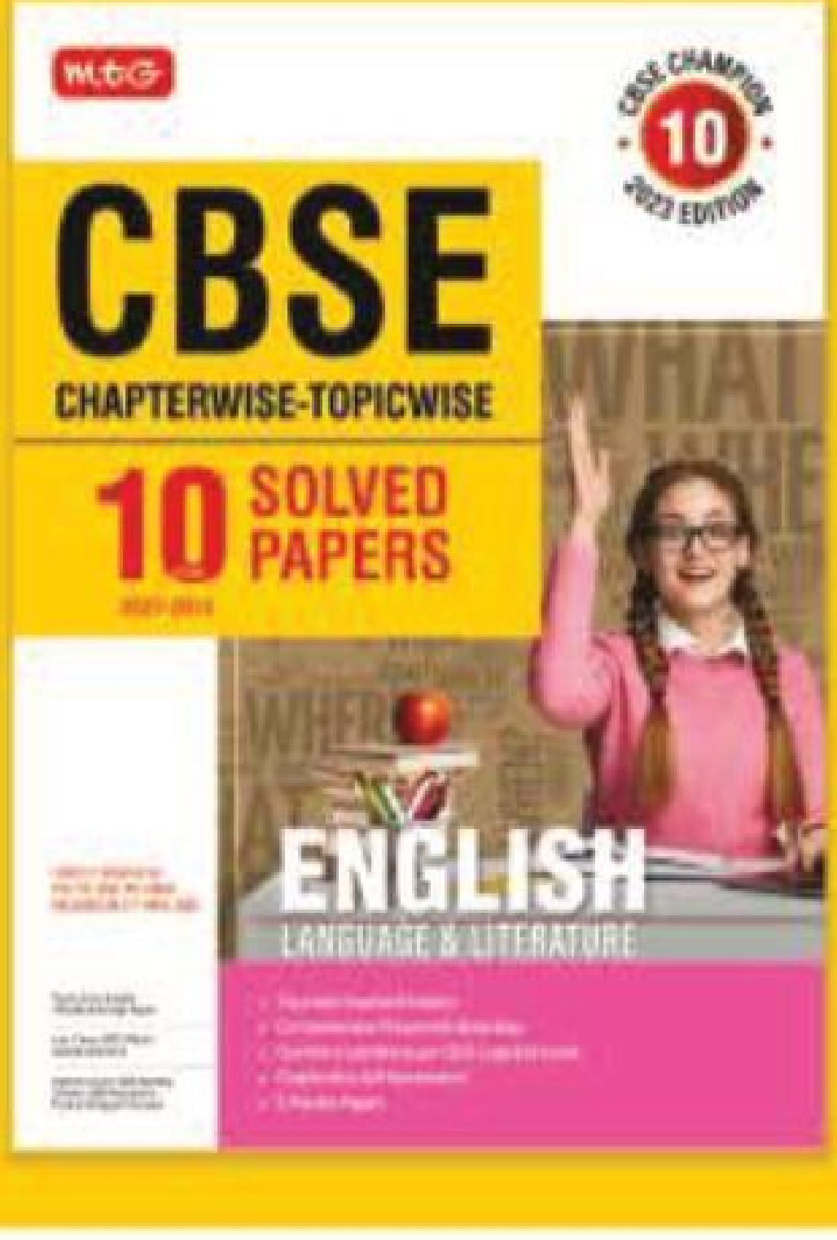
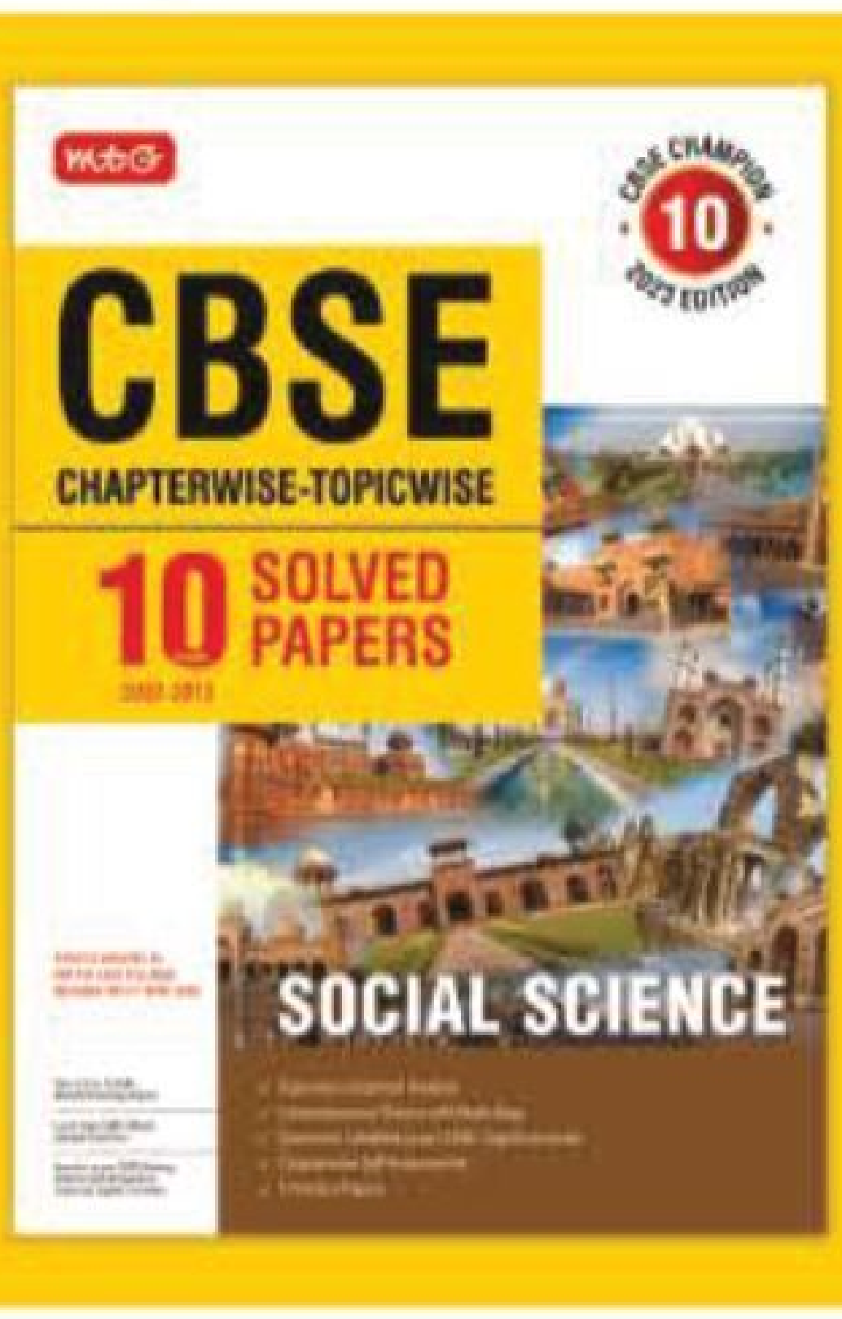
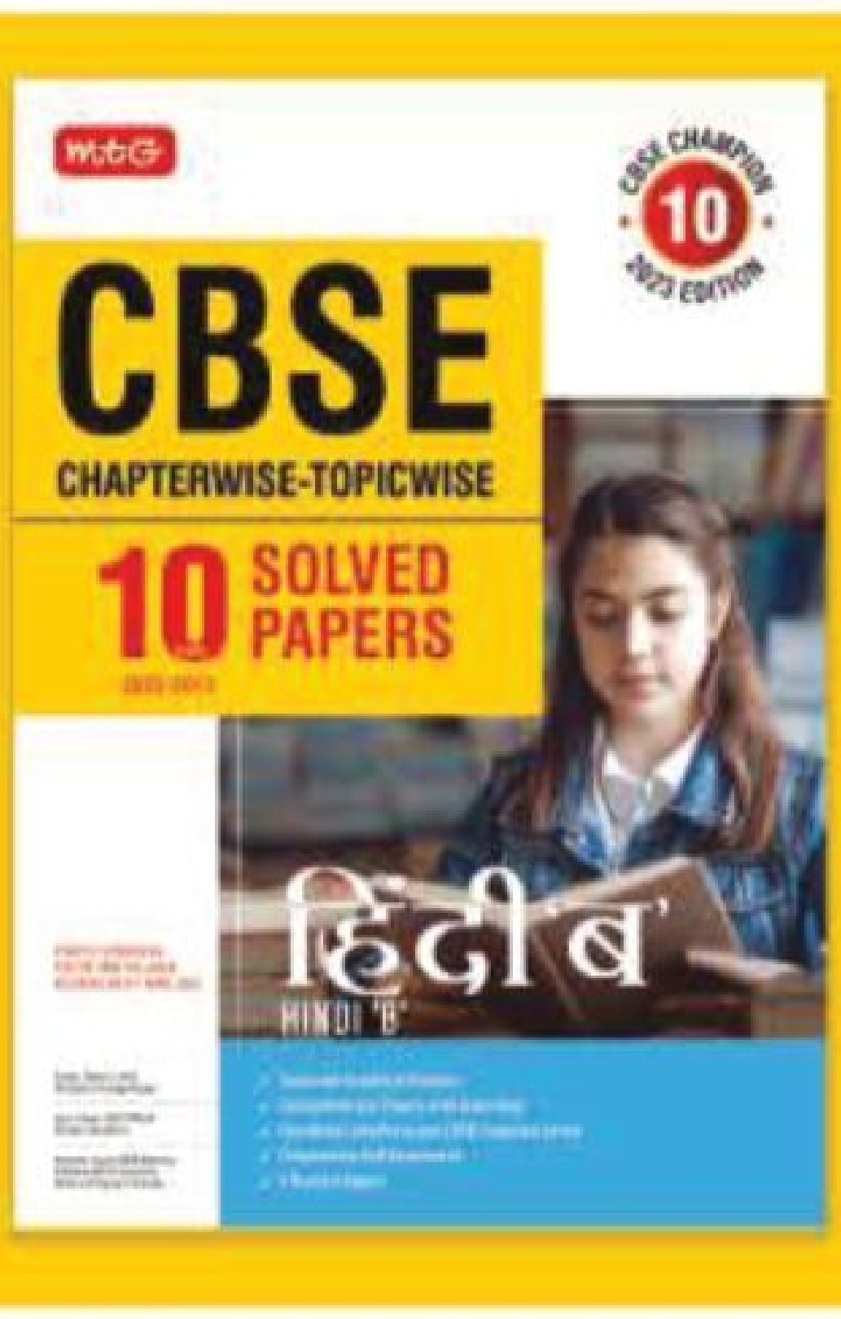

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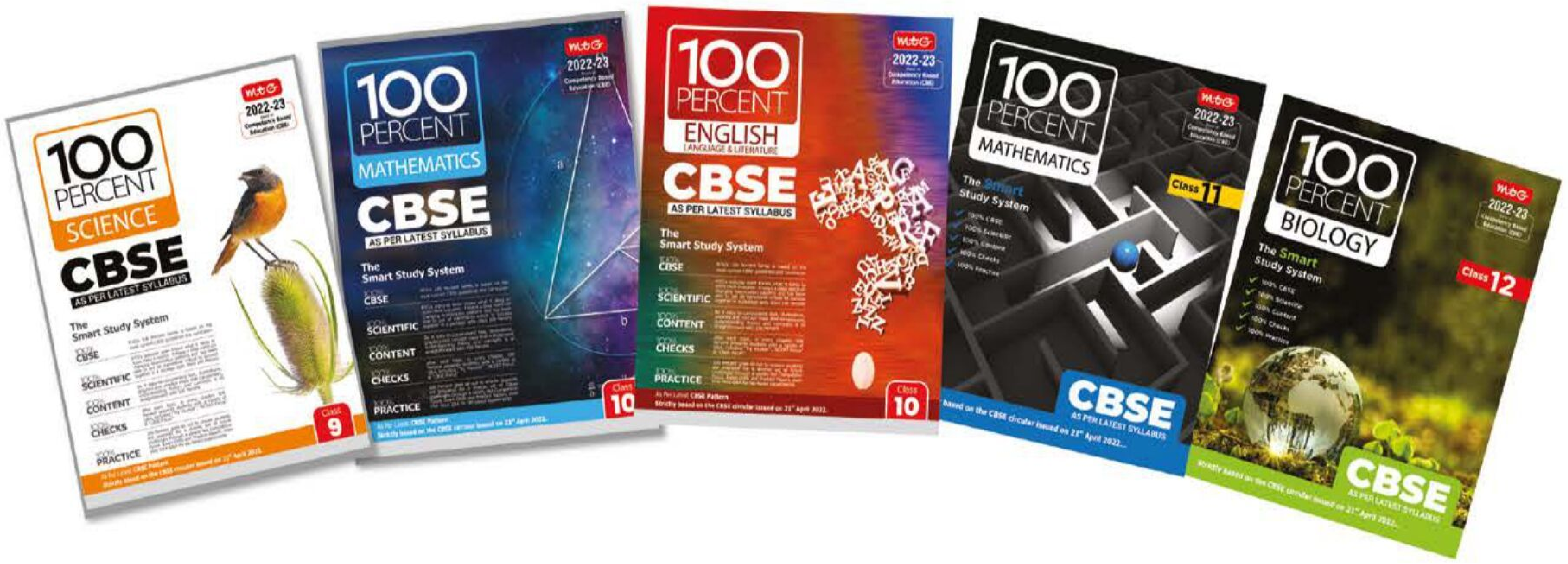

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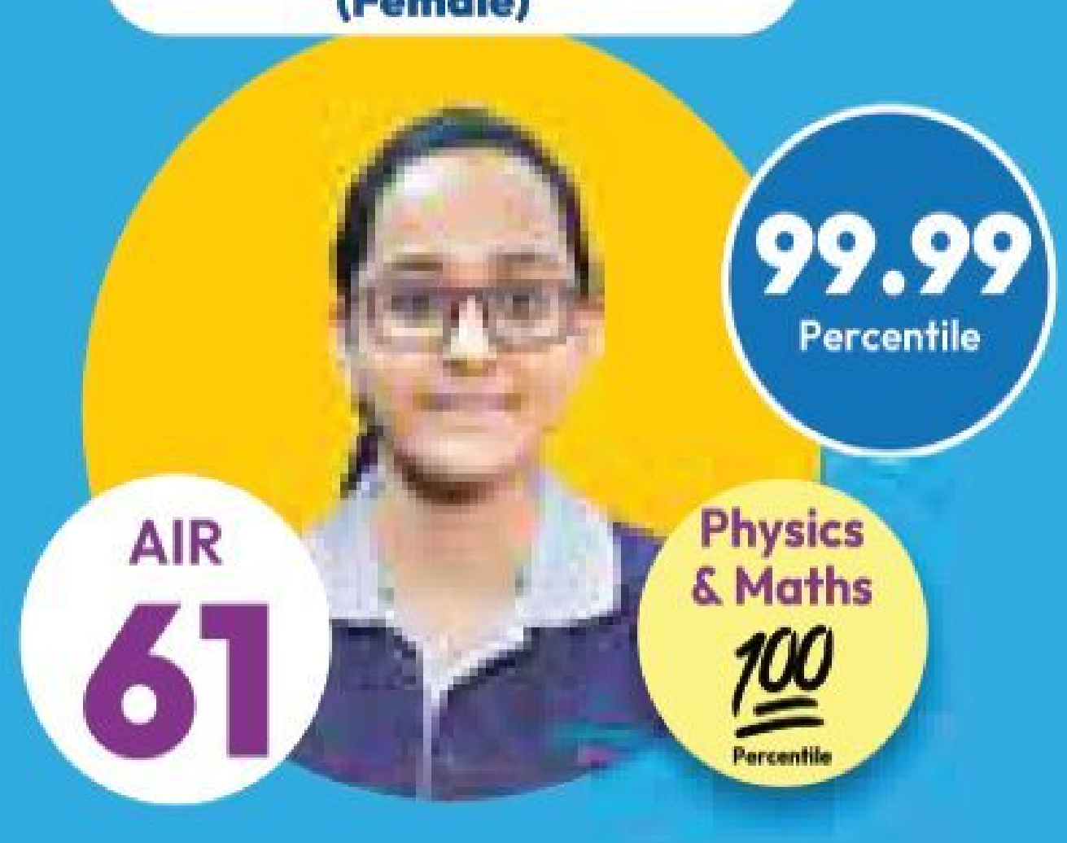
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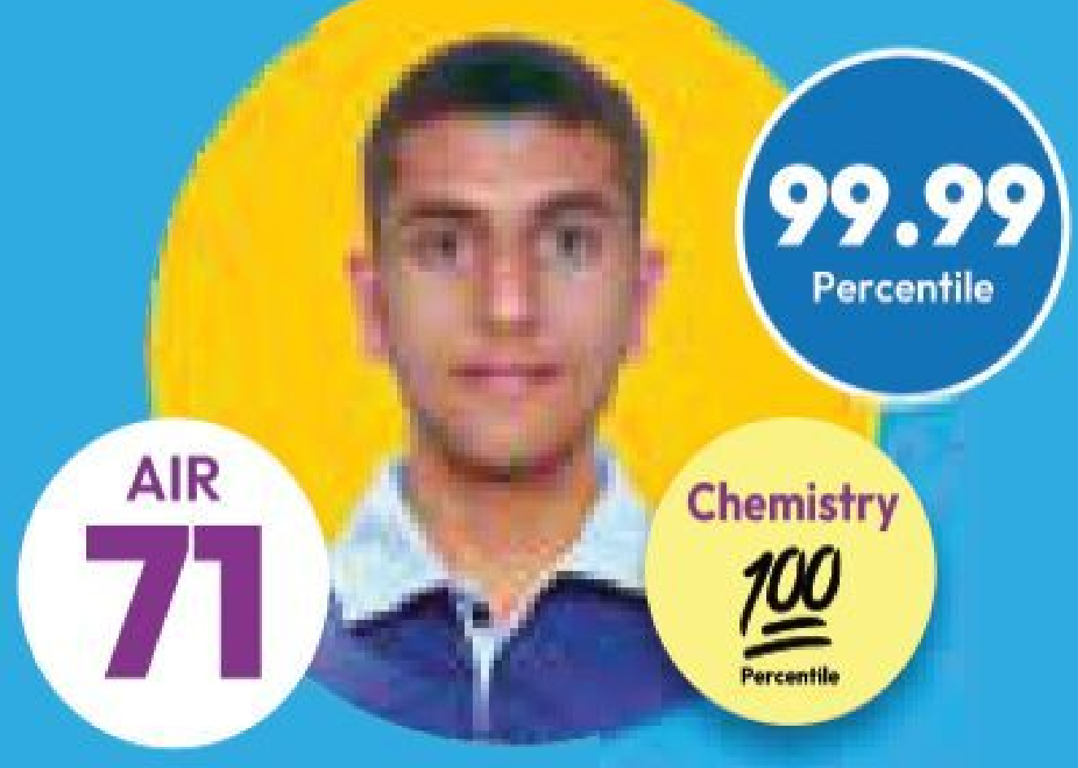
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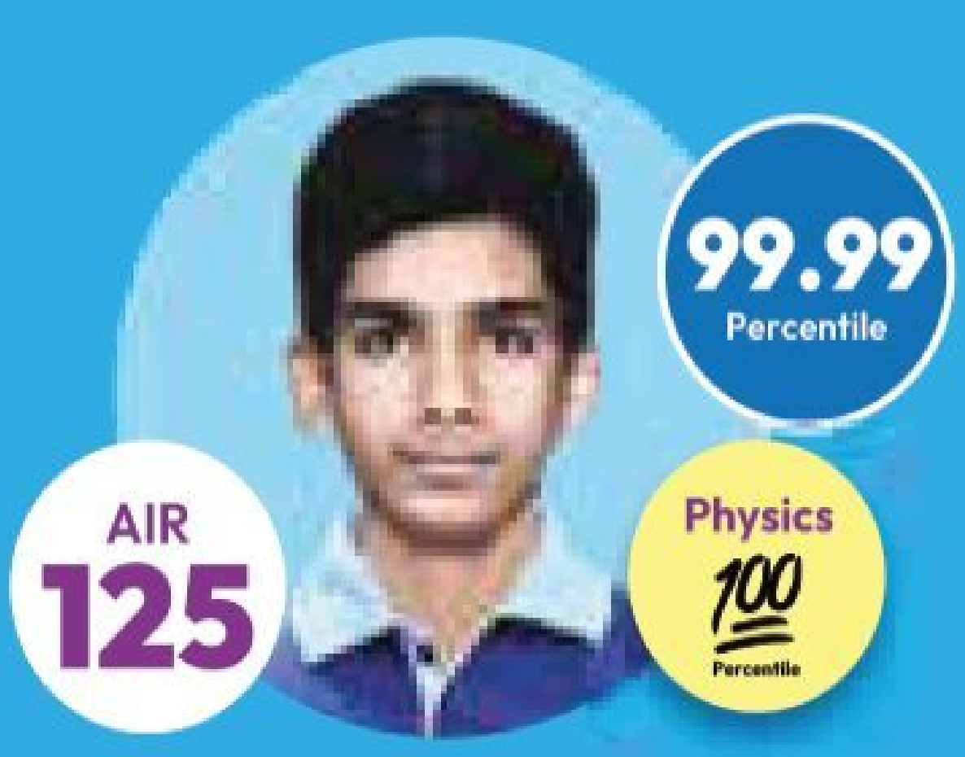
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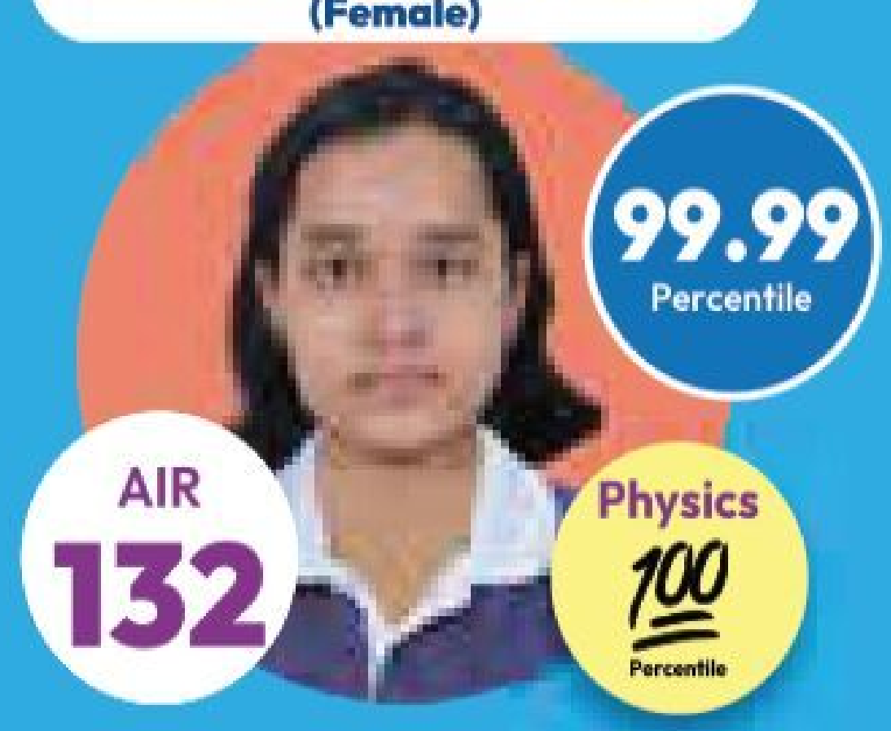
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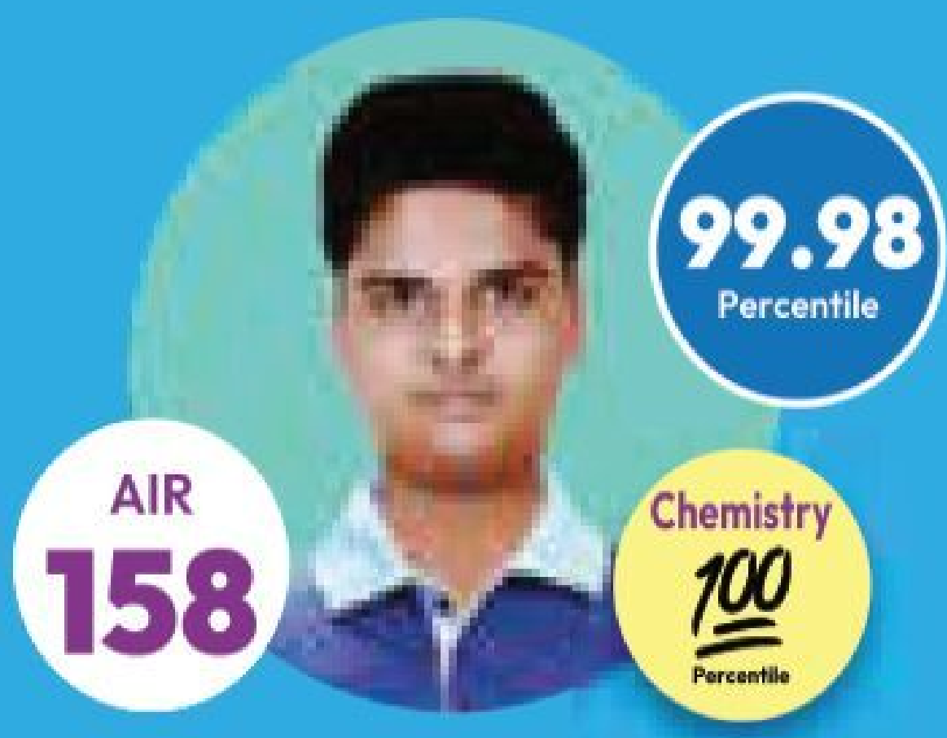
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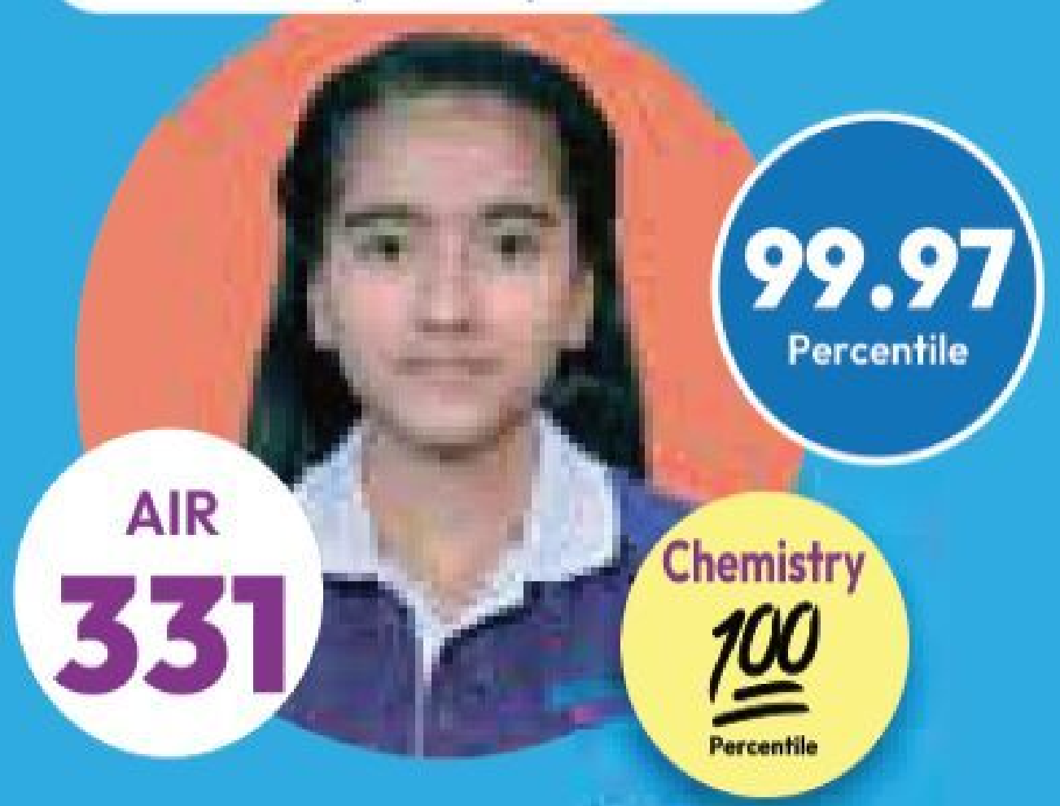
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